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MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

A New Absolute Determination of the Acceleration due to Gravity at the National Physical Laboratory, England

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A NEW ABSOLUTE DETERMINATION OF THE ACCELERATION DUE TO GRAVITY AT THE NATIONAL PHYSICAL LABORATORY, ENGLAND

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[Plate 17]

CONTENTS

A new absolute determination of the acceleration due to gravity at the National Physical Laboratory has been made by timing the symmetrical free motion of a body moving under the attraction of gravity; it is the first time this method has been used. The moving body was a glass ball and it was timed at its passage across two horizontal planes by the flashes of light that it produced when it passed between two horizontal slits which served to define each plane optically, the ball focusing light from one of the slits, which was illuminated, upon the other slit which had a photomultiplier placed behind it. The separation of the two planes defined by the pairs of slits was measured interferometrically and referred directly to the international wavelength definition of the metre, while the time intervals were measured in terms of the atomic unit of time scale A1. The value of gravity as reduced to the British Fundamental Gravity Station in the N.P.L. is

$$981\,181{\cdot}75\ \text{mgal},\quad \text{s.d.}\ 0{\cdot}13\ \text{mgal}\quad (1\ \text{mgal} = 10^{-5}\,\text{m/s}^2).$$

Systematic errors, are believed to be very small; this is particularly true of the error due to air resistance. The main contribution to the observed scatter of the results comes from microseismic disturbances.

The new result is 1·4 mgal less than that obtained at the fundamental station by J. S. Clark

(1939) using a reversible pendulum. It is very close to the mean of a number of recent absolute determinations by other methods, but this may not be very significant because the uncertainties of those determinations and of the comparisons between the sites at which they were made and the present site are not less than 5 times the standard deviation of the new result.

1. Introduction

This is an account of a new absolute determination of the acceleration due to gravity by a symmetrical free motion method involving the direct measurement of the acceleration of a body moving freely up and down under the gravitational attraction of the Earth; the results are the first to be obtained by the method. The reasons for making accurate measurements of the acceleration due to gravity and the characteristic features of the methods that are available, recently discussed in a review article (Cook 1965*b*), can be summarized very briefly. Accurate measurements of the acceleration due to gravity in absolute terms are needed to realize forces in absolute terms so that quantities such as pressure and the ampere may be reproduced consistently in different places and at different times; the mean absolute value of gravity over the Earth enters the system of fundamental constants that describe the dynamics of the solar system (Cook 1965*a*); accurate absolute measurements would help to control worldwide measurements of differences of gravity; lastly, if repeated at intervals of a few years, they might serve to detect possible secular changes of gravity. For all these purposes, accuracies of one part in a million are necessary, and it would be very desirable to achieve one part in ten million.

Many measurements have been made in recent years, either with reversible pendulums or by timing a body falling freely under gravity, the latter method being quite distinct from that described in this paper in which the body is timed over the upward as well as the downward part of its path and not just the downward part. Pendulum and free fall determinations are less satisfactory in principle than a symmetrical free motion determination, the former on account of uncertainties in the behaviour of a knife-edge support, the latter because of possible systematic errors in the timing, poor combination of the observations and the relatively large effects of air resistance; thus, when it was decided to undertake a new absolute determination at the N.P.L. it seemed worth while to attempt a symmetrical free motion experiment which might be of greater technical difficulty but is, of all methods so far proposed, the least subject to systematic errors. The review article mentioned contains a detailed discussion and here it is sufficient merely to list the particular advantages of the symmetrical free motion method. The principle, first suggested by Volet (1947), is indicated in figure 1. An object is thrown up vertically and is timed at its successive crossings of two horizontal planes separated by a distance H. Let the time between the two crossings of the lower plane be T_a and that between the two crossings of the upper plane be T_b. If the acceleration of the body is constant, it is equal to

$$8H/(T_a^2 - T_b^2).$$

In a free fall experiment, the time intervals required are those between a passage across a plane near the top of the drop and another across a plane near the bottom of the drop where the speed of the body is greater and the duration of the signal less than for the upper passage. Consequently there may be a systematic error in the measured time interval if the time constants of the timing systems are not carefully chosen. In a symmetrical free motion experiment, on the other hand, the time intervals are between passages across the same planes and therefore at the same velocities so that the durations of the two signals that define a time interval are the same; in consequence the time constants of the apparatus are only required to remain unchanged during the flight of the object.

The deceleration of a glass ball of 2 cm diameter falling at 5 m/s (its velocity after falling 1 m from rest) due to air at a pressure of 1 μb† is 24×10^{-5} m/s² so that to obtain results reliable to 1 mgal‡ without applying a correction for air resistance, the air pressure must be less than 0·01 μb. In various experiments, objects of more complex form and greater density have been used, the resistive deceleration of which cannot be calculated exactly;

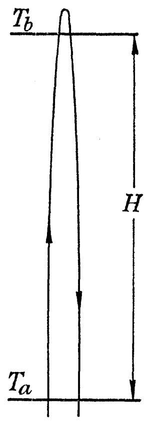

FIGURE 1. Principle of symmetrical free motion determination.

it is not easy to set limits to the pressure that should be achieved but certainly it should not be greater than 0·01 μb. The symmetrical free-motion scheme has the remarkable property that the measured acceleration is independent of resistive forces to first order, provided that those forces are proportional to the velocity of the moving body and not to any other power, although for other power laws, the error in the measured acceleration would be somewhat less than in a free fall experiment. In the present work, the prediction has been verified over a very wide range of air pressure, demonstrating that over that range the resistance is in fact proportional to velocity, so that it was possible to obtain reliable results at pressures of air very much greater than are permissible in a free fall experiment. In general this might not matter very much since the pressures in question are not difficult to achieve, but in the present work in which a glass ball was the moving object, considerable trouble was experienced with electrostatic charging of the ball at the lowest pressures and it was very convenient to be able to work at higher pressures.

The acceleration due to gravity varies by about 5 gal or 1 in 200 between the equator and the poles, and the value at any particular place is strongly affected by quite local factors, the height of the site above sea level and the densities of the rocks in the neighbourhood. For the realization of standards at a particular laboratory, it is the arbitrary value of gravity

† 1 b = 10^5 N/m², 1 μb = 0·1 N/m².
‡ 1 mgal = 10^{-5} m/s²; it is the unit of acceleration commonly employed in geophysics and will be used here.

at that laboratory that is required, but for other purposes that value must be related to others at many different sites. Furthermore, while many absolute measurements of gravity have been made in recent years, it is exceptional for different methods to be used at any one site (the N.P.L. is one of the few exceptions) and therefore to enable results of different absolute determinations to be compared, the differences of gravity between the sites must be well known. This presents many difficulties and will be dealt with in only a preliminary way in this paper.

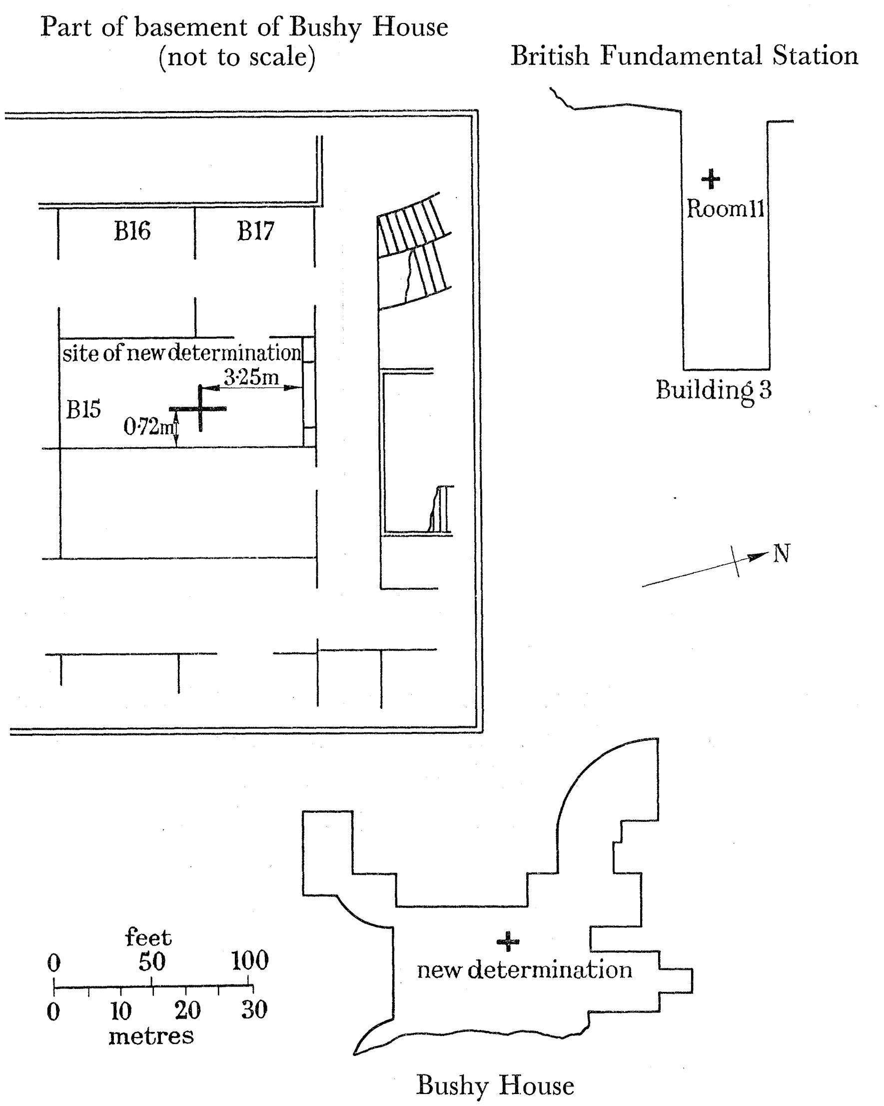

FIGURE 2. Sites of new determination and British fundamental gravity station.

To permit comparisons between determinations to be made with an accuracy of 0·1 mgal, the position at which gravity is measured must be carefully defined. The present measurement was made in a basement room (15) of Bushy House, one of the buildings of the N.P.L. The geographical coordinates of the site are: latitude, N. 51° 25′ 11″·9; longitude, W. 00° 20′ 15″·6; height of floor 8·30 m (Newlyn datum).

These are of course inadequate to identify the site and a diagram of the room in relation to Bushy House and to the British Fundamental Gravity Station (*Nature, Lond.* **173** (1954),

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794) is given in figure 2. Local changes have been measured with a spring balance gravity meter with the following results: change of gravity with height—value at floor level, 0·000 mgal; at 76 cm, −0·243; at 154 cm, −0·476.

The difference of gravity at floor level from that at the Fundamental Station has been measured on two occasions.

Gravity at floor level − gravity at fundamental station: 1958, +0·05 mgal; 1962, +0·09 mgal; mean, +0·07 mgal, s.d. 0·02 mgal.

These results enable the present measurements to be related to the value of gravity at the fundamental site, which is on the pillar on which the previous absolute determination at N.P.L. was made (Clark 1939).

2. Principles of the method

The design of the experiment turns on the way in which a signal is obtained from the moving object as it crosses the reference planes. Free fall experiments with rods had shown that (see Cook 1965*b*) it was very difficult to release a rod from rest so that it fell vertically without rotating and had also shown that rotations and deviations from vertical motion could cause errors in the measured acceleration. The difficulties in throwing a rod up were likely to be yet greater and it seemed best to adopt a method that would be independent of rotations of the object. A glass ball is a very poor lens, but it was found that the central part of an image of a slit formed by a glass ball is quite well defined and that a practical method of deriving a signal is to define a plane by two parallel horizontal slits at such a distance apart that a glass ball midway between them will focus the one on the other. The proper separation is about 3 times the diameter of the ball when the refractive index is 1·5. One of the slits is illuminated and a photomultiplier is placed behind the other, and then as the ball passes between the slits at right angles to the plane that they define, the photomultiplier emits a pulse of current that is only slightly degraded from the pulse that would be obtained with an ideal lens. The actual form of the pulse with slits nominally 5 μm wide is shown in figure 3. If the ball is perfectly spherical, and if the density and refractive index are uniform, then clearly the time at which the peak of the signal occurs will not depend on the rotation of the ball and, in fact, it is not difficult to obtain balls that satisfy these conditions. The essential requirement is that the optical centre of the ball should always maintain the same position relative to the centre of mass, which is achieved if the ball is spherical and uniform; in practice, it was found that the method of throwing the ball up imparted very little rotation to it.

Horizontal movements of the ball in the plane of the slits may have three effects: they may cause errors in timing owing to geometrical misalinement, they may degrade the condition for focus, and they may reduce the amplitude of the signal. The latter two, which are connected with the optical properties of the ball and slit system, are discussed in the next section, while the purely geometrical effect is considered here.

The peak of the optical signal occurs at a vertical position of the ball relative to a pair of slits which is hard to determine with precision. In the first place, the position of the slits themselves is not well defined. They are formed by ruling a line with a diamond through a thin layer of aluminium deposited on a glass block, as described in the next section and, on

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account of diffraction effects, the effective position of such a slit viewed under a high power microscope depends on the conditions of illumination. Furthermore, the position of the ball at which the maximum signal is obtained depends on the distribution of the light passing through the slits, and again diffraction effects are involved especially if, as is quite possible, the aluminium is not entirely removed over the width of a slit. For these reasons the effective position of the horizontal plane defined by a pair of slits must be regarded as unknown and the experiment must be arranged to eliminate it from the calculations of the acceleration. The glass blocks carrying the slits were therefore held by molecular adhesion between pairs of glass slabs with optically flat parallel sides (having holes for the ball to pass through), the whole forming a composite block with plane parallel horizontal faces parallel to the

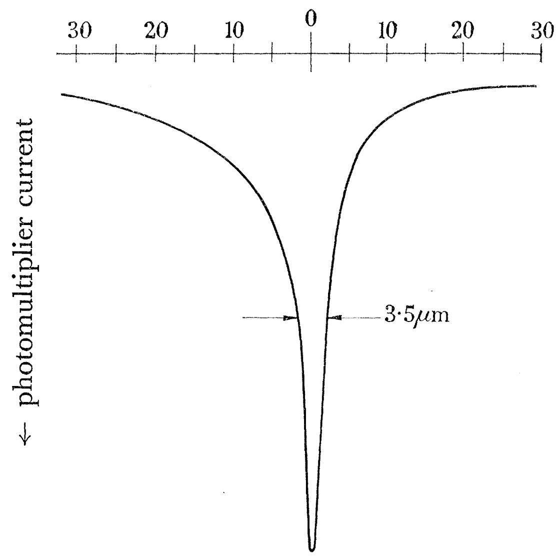

FIGURE 3. Form of photomultiplier pulse from glass ball between pair of slits.

effective plane of the slits (figure 4 and figure 5, plate 17). One such block was at the bottom of the trajectory of the ball and one at the top, and the separation of the blocks could be determined by interferometric measurements between the opposing faces of the two blocks. Let this distance be D_1 and let the distances of the effective planes of the slits from the upper face of the lower block and from the lower face of the upper block be x_a and x_b respectively. Then the height H_1 of the upper slit plane above the lower is

$$H_1 = D_1 + x_a + x_b \quad \text{(figure 6)}.$$

If both blocks are turned over so that the distances of the slit plane from the faces between which the separation is measured are $t_a - x_a$ and $t_b - x_b$, where t_a and t_b are the thicknesses of the lower and upper blocks respectively, then

$$H_2 = D_2 + t_a - x_a + t_b - x_b.$$

But H is equal to $\frac{1}{8}g(T_a^2 - T_b^2)$ and so

$$\tfrac{1}{8}g[(T_a^2 - T_b^2)_1 + (T_a^2 - T_b^2)_2] = D_1 + D_2 + t_a + t_b,$$

enabling g to be determined without knowing the effective positions of the slit planes within the blocks. In fact, overturning the blocks gives four possible configurations and x_a and x_b can be found as well as g. The blocks were also turned about a vertical axis and were interchanged top to bottom so that the results were derived from three sets, each of four configurations and the value of gravity depends on twelve sets of time and length measurements.

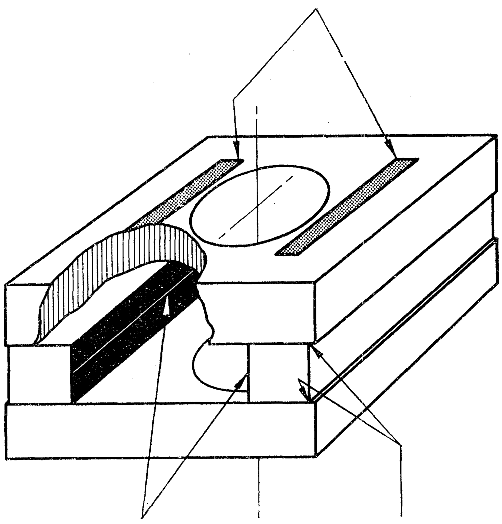

FIGURE 4. Diagram of one slit block assembly.

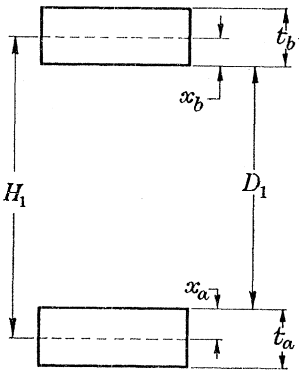

FIGURE 6. Principle of inversion of slit blocks.

Because of the symmetry of this scheme, the only geometrical error of any significance is that arising from the surfaces of the blocks not being horizontal. Let the separations of the blocks (figure 7) be measured at points that are distant w from the nominal trajectory of the ball (w is between 3 and 5 cm) and let the distances be D_{11} and D_{12}. Let the thicknesses of the lower block at these positions be t_{a1} and t_{a2} and those of the upper block be t_{b1} and t_{b2}. The positions of the slit planes are defined by the distances x_{a1}, x_{a2}, x_{b1} and x_{b2} and the upper surface of the lower block is inclined at an angle α to the horizontal. The trajectory of the ball is such that it passes midway between the two lower slits on its upward flight but when it falls back to this level it is displaced a distance $2y$ from the centre.

If the ball does not fly in a truly vertical trajectory and if the slit plane is not truly horizontal, then the total time spent above the plane is, to first order, equal to the time for a vertical trajectory through the apex of the actual trajectory. The second order correction is of the order of $(\alpha y/D)^2$ and is negligible in practice. It is therefore sufficient to calculate the

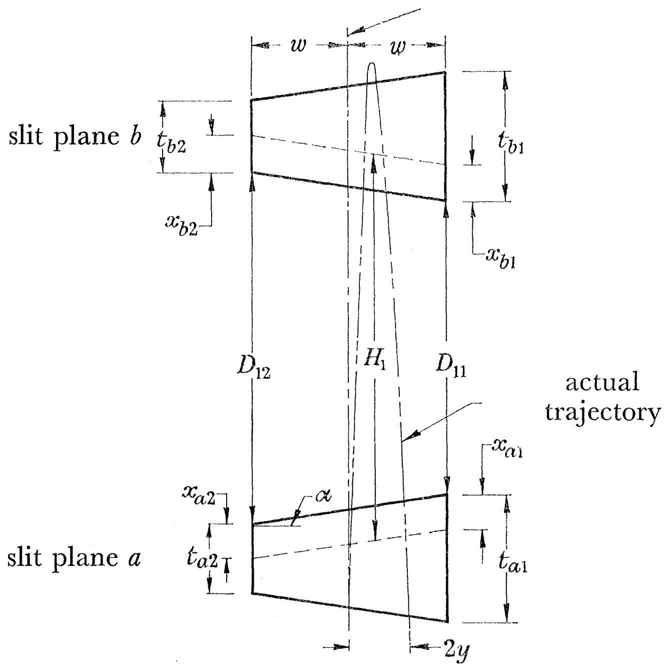

FIGURE 7. Detailed geometry of the slit block measurements.

vertical trajectories through the actual vertices for the conditions shown in the figure. The height H_1 is

$$\tfrac{1}{2}\{D_{11}+D_{12}+x_{a1}+x_{a2}+x_{b1}+x_{b2}\}\cos\phi_1+y\{D_{11}-D_{12}+x_{a1}-x_{a2}-x_{b1}+x_{b2}\}/2w,$$

where ϕ_1 is the mean angle $\{\alpha_1+\tfrac{1}{2}(D_{11}-D_{12})/w\}$.

On inverting the lower block, x_{a1} is replaced by $(t_{a1}-x_{a1})$ and x_{a2} by $(t_{a2}-x_{a2})$ and α will also change. y is different for each flight of the ball and w also differs for each configuration because, as will be seen in the next section, the positions at which the interferometric measurements are made are different in each configuration; it is, however, straightforward to reduce all the interferometric measurements to the same value of w.

Now average over the four configurations and in each take mean values of y since a number of flights is timed in each configuration. The mean height H_m is then

$$\tfrac{1}{2}\{D_{m1}+D_{m2}+\tfrac{1}{2}(t_{a1}+t_{a2}+t_{b1}+t_{b2})\}$$
$$-\tfrac{1}{2}(D/w^2)\,\langle\{w\alpha_i+\tfrac{1}{2}(D_{i1}-D_{i2})\}^2\rangle_{\text{av.}}+\langle y_i(D_{i1}-D_{i2})/2w\rangle_{\text{av.}}$$

The first term corresponds to exact alinement. The second is the cosine error that arises because the surfaces of the blocks are not horizontal and since it is a small term, the mean value, D, of all the measured values of the separation has been used. $(D_{i1}-D_{i2})$ never exceeded $1\cdot0\,\mu$m and rarely exceeded $0\cdot25\,\mu$m, the corresponding values of $\frac{1}{2}(D_{i1}-D_{i2})/w$ being 2 and $0\cdot5\times10^{-4}$. α was always adjusted to be less than 10″ (5×10^{-5} rad) and the cosine error may therefore be taken to be less than $5\times10^{-8}D$. It is of course due to the whole apparatus not being correctly alined relative to the direction of gravity and the only error due to internal misalinement of the apparatus is that given by the third term, in which an error of parallelism of the surfaces of the blocks interacts with a departure of the trajectory of the ball from the vertical. This may be systematically in one direction, but y cannot exceed 0·5 mm without the optical system failing to work and since $(D_{i1}-D_{i2})$ is less than $1\,\mu$m, it can be seen that the error should not exceed $0\cdot01\,\mu$m.

To summarize this analysis, errors due to misalinement may be taken to be less than 1 in 10^7. Detailed estimates are given later.

The reversal method for the elimination of the slit position depends on the values of x and t remaining constant throughout a set of measurements and leaving aside any secular changes, which will be seen to have been very small, the effects of temperature changes must be considered. The four observation equations of a set may be written:

$$H_1 = D_1 + x_a(1+\alpha\theta_{a1}) + x_b(1+\alpha\theta_{b1}),$$

$$H_2 = D_2 + (t_a-x_a)\,(1+\alpha\theta_{a1}) + (t_b-x_b)\,(1+\alpha\theta_{b2}),$$

with two similar equations; θ denotes the temperature and α is the coefficient of expansion of the blocks. Averaging over the four positions of the blocks, we have

$$H_m = D_m + \tfrac{1}{2}t_a(1+\alpha\theta_{am}) + \tfrac{1}{2}t_b(1+\alpha\theta_{bm}) + \tfrac{1}{4}\xi_1\alpha(\theta_{a1}-\theta_{a2}+\theta_{a3}-\theta_{a4}) + \tfrac{1}{4}\xi_2\alpha(\theta_{b1}+\theta_{b2}-\theta_{b3}-\theta_{b4}),$$

where ξ has been written for $(\frac{1}{2}t-x)$.

The terms involving t_a and t_b are known, ξ is less than $100\,\mu$m, α is $0\cdot5\times10^{-6}$/degC for fused silica, the variation of the temperatures does not exceed 1 degC, and so the error term is quite negligible.

The arrangement of the experiment will now be clear. In each of the four configurations of the slit blocks, a number of measurements is made of the pairs of time intervals, T_a and T_b and the separation of the blocks is measured interferometrically. The thicknesses of the blocks are found by separate interferometric measurements. Subsequent sections contain detailed descriptions of the construction and measurement of the slit blocks (§3), the interferometer system (§4), the timing system (§5), the vacuum system and the catapult for shooting up the ball (§6). The effects of electrostatic charging and the dependence of the measured acceleration on the pressure of residual air are also discussed in detail (§7, 8), as are the measurements of microseisms and their effects (§9).

The value of gravity decreases with height by 0·3 mgal/m; since the separation of the slits is very nearly 1 m, it is necessary to consider the precise level to which the measured acceleration is referred. If $g_{\text{obs.}}$ is the value of the acceleration calculated from the formula $8H/(T_a^2-T_b^2)$, then the value of gravity at the height of the lower slit is

$$g_1 = g_{\text{obs.}}\{1+\tfrac{1}{12}\gamma T_a^2-\tfrac{1}{24}\gamma T_b^2\},$$

where γ is the rate of change of gravity with height (Cook 1965*a*).

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3. The slit blocks

The construction of the slit blocks is indicated in figure 4 and was briefly described in the preceding section. Two sets of blocks have been made, the first of glass for preliminary work and the second of fused silica for the final determinations. There were three reasons for replacing the glass by the silica blocks. First, the slits ruled on the glass blocks were not completely clear because they were formed with a diamond with a very narrow edge and it was necessary to make five parallel strokes to produce a slit with a width of about 5 μm that preliminary experiments had shown gave the sharpest signal with a ball of 2·5 cm diameter (the wider angle through which light was diffracted by a narrower slit gave a less well defined signal). Subsequently, a broad edge was formed on a diamond and it was possible to rule a satisfactory slit with one stroke. Secondly, the rather large coefficient of thermal expansion of glass gave rise to a slight uncertainty in the estimated thickness of a block which was quite absent with fused silica. Thirdly, fused silica can be worked more accurately and assembled more securely by molecular adhesion than can glass.

The slits were ruled parallel to the worked faces of a block to within 1 μm over the length of 8 cm, and the slits on the two blocks belonging to the same assembly were at the same distance from corresponding faces to within the same limit so that the plane defined by the slits is parallel to the worked outer faces of the composite block to within about 1 μm in 8 cm or so in the direction parallel to the slits as well as in the horizontal direction perpendicular to the slits.

The optical performance of the slit and ball system depends critically on the separation of the slits. If the refractive index of the ball is 1·5 exactly, then the separation of the slits corresponding to the minimum conjugate object-image distance should be $3D$ where D is the diameter of the ball, but for the actual balls of slightly different refractive index the best separation was found experimentally and had to be maintained to within 0·03 mm in the assembly of the blocks. If the slit separation is not correct, not only is the image worse when the ball is midway between the slits, but the focus deteriorates much more rapidly when the ball is away from the centre; the quality of the image would then be sensitive to the horizontal position of the ball.

The intensity of the light transmitted by the assembled slit pairs, as a function of the vertical position of the ball, is shown in figure 3. Such curves were obtained statically but those observed with a ball in motion are consistent with them. It should be possible to determine the peak of the signal to the equivalent of about 0·2 μm and this expectation was borne out in practice. Unfortunately one of the four slits was not ruled as clearly as the other three and the assembly in which it was incorporated therefore gave a less intense signal than the other, leading to some unreliability in the operation of the timing system but not to any loss of accuracy when the system did operate.

It was most important to eliminate as thoroughly as possible stray light passing through the slit system, for the signal has to be detected in the presence of such light. The most serious difficulty comes from noise pulses that are large enough to operate the timing system and which are very troublesome even though they occur only once every 10 or 20 s. The rate at which pulses of a given height occur is proportional to the square root of the intensity of the stray light, while the amplitude of the wanted signal is proportional to the

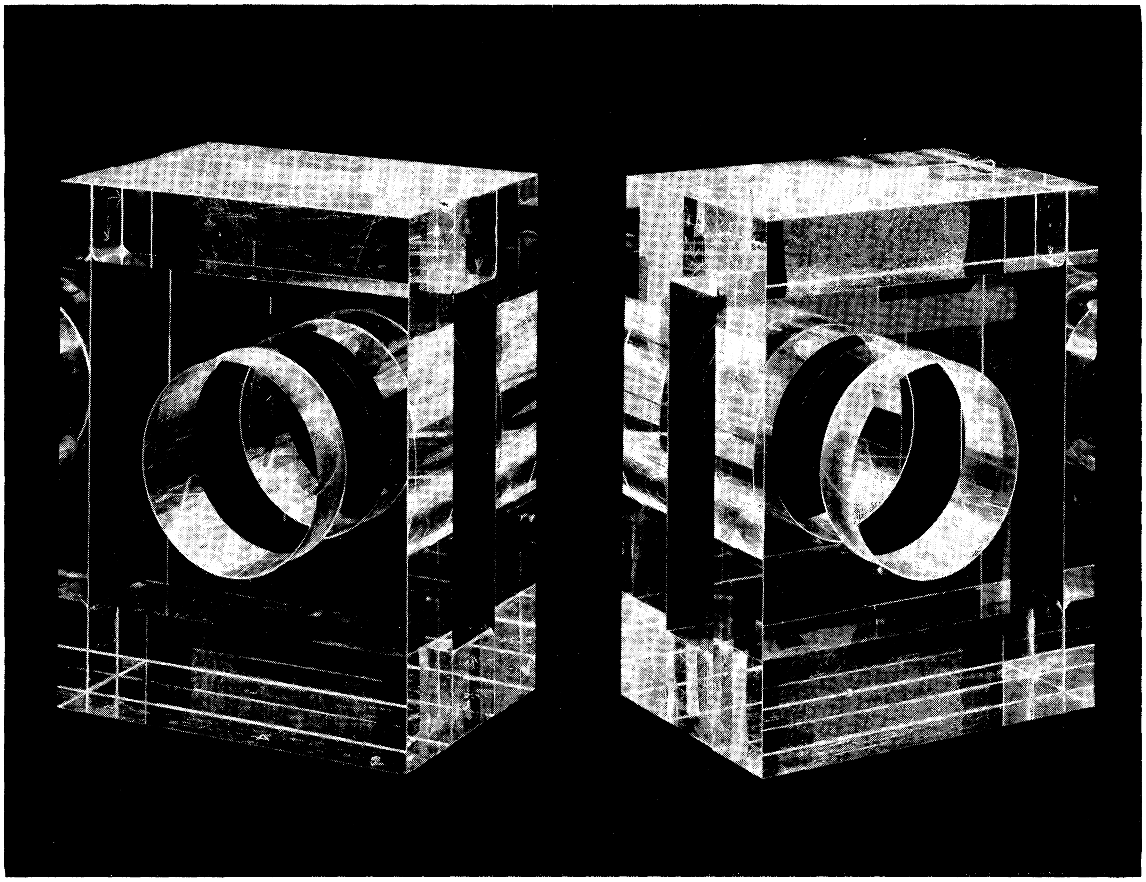

FIGURE 5. Photograph of slit blocks.

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intensity of the light focused by the ball. The stray light was cut down as much as possible by careful masking and in particular by restricting the operative length of the slits; at the same time the intensity of the light by which the slit is illuminated was kept as high as possible and a strip filament lamp was used with the slits that had the lower transmission. A consequence of restricting the effective length of the slits is that the trajectory of the ball must be alined just as accurately in the direction along the slits as in the direction between them. Noise on the actual signal was never important.

The outer horizontal faces of the assemblies were coated with a semi-transparent layer of aluminium in order to enhance the fringes in white light used for the interferometric measurements of distance. Since each face of an assembly is used in turn as an interferometric surface and is used with each face of the other assembly, the coatings were applied in patterns such that in each configuration of the blocks, there was a path for the white light through uncoated parts of the outer surfaces and coated parts of the inner surfaces. For each configuration there were two paths symmetrical about the centre of the block.

The optical thickness of a block was measured by comparison with a Fabry–Perot interferometer of fused silica, as shown in figure 8. Although the optical thicknesses of the coatings are included in the measured thickness, they also enter the white light interferometer measurements, and are eliminated from the final mean separation of the slits.

The Fabry–Perot interferometer was made of four blocks of optically worked fused silica in molecular contact, the space they enclosed being large enough for a slit block to be placed inside. Appropriate parts of the interferometer faces were coated with semi-transparent films of aluminium. The optical length was determined quite conventionally by illuminating the interferometer with light from a lamp containing mercury-198 and photographing the interference fringes formed in the various wavelengths through a prism spectrometer. The results, given in table 1, show that the interferometer was very stable.

TABLE 1. LENGTH OF THE FUSED SILICA INTERFEROMETER

(Measurements with mercury-198)

date	number of wavelengths	mean temperature (°C)	length at 20 °C (μm)
June 1960	4	20·1	73880·500
June 1964	5	19·4	0·495
July 1965	6	19·3	0·500

Adopted mean value: 73880·498 μm, s.d. 0·008 on 2 degrees of freedom.

In the optical system for the comparison of a slit block with the Fabry–Perot interferometer (figure 8), a prism supported inside the slit block reflected light from a lamp to the two faces of the block. The block was so placed in the interferometer that an outer face of the block and the corresponding inner face of the interferometer formed a Fabry–Perot interferometer, the fringes from which were viewed in telescopes provided with micrometer eyepieces. Filters selected the wavelengths. Two observers measured the diameters of the fringes on the two sides simultaneously. It was essential to measure simultaneously: the slit block and the interferometer were on separate levelling tables and changes of temperature of the apparatus led to movements of the one relative to the other so that the two gaps between the block and interferometer changed in opposite directions although the sum of the two remained unaffected. The sum of the two gaps was found from the sums of the fractional orders of interference on the two sides in the various wavelengths.

The results are summarized in table 2. It will be seen that one of the blocks is not quite parallel. The changes with time, which are not large enough to cause any significant uncertainty in the measured value of gravity, no doubt arise in part from the strain imposed on the silica blocks by assembling them.

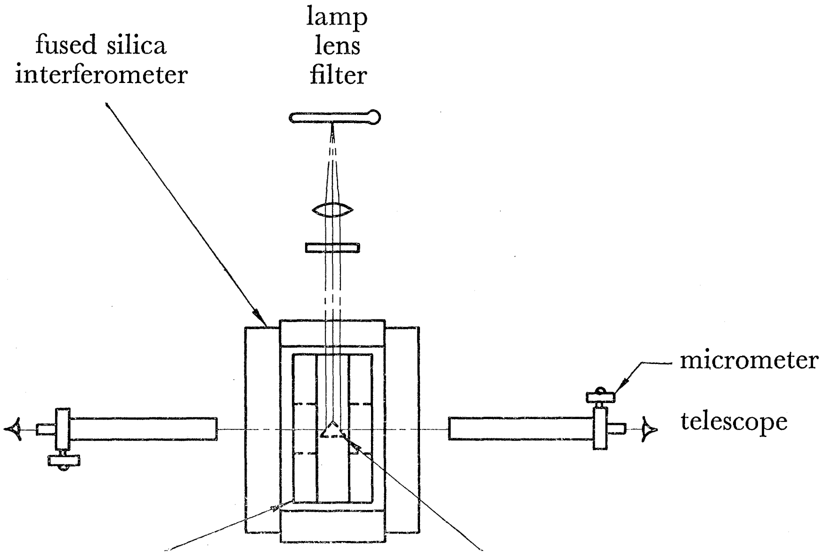

FIGURE 8. Plan of system for measurement of thickness of slit block.

TABLE 2. THICKNESSES OF THE SLIT BLOCKS

(Lengths at 20 °C (μm).)

	block *A*		block *B*	
date	side α	side β	side α	side β
Nov. 1964	57098·082	57098·114	57114·236	57114·252
July 1965	57098·150	57098·161	57114·109	57114·273
mean	57098·116	57098·138	57114·172	57114·262

mean: block *A* 57098·127, block *B* 57114·217 } s.d. 0·027 μm on 4 degrees of freedom.

both blocks 57106·172 s.d. 0·019 μm.

Standard deviations derived from differences between the measurements on the two occasions.
Including the uncertainty of the fused silica interferometer, the s.d. of the mean of both blocks is 0·021 μm on 4 degrees of freedom.

The outer horizontal faces of the assembled blocks are appreciably curved, another consequence of the strain imposed on assembly.

The measured thickness of a block is the distance between the coating on the one face and that on the other, the measurements being made by reference to the flat and parallel faces of the outer fused silica interferometer, and this is the correct value to use in combination with the measured separation of the blocks. The curvature does not cause a serious error, for although it is true that in different configurations of the blocks, different parts of the semi-reflecting strips are used, it is found that the mean thickness measured over the whole width of the strip is within 0·01 μm of the value that is correct for either section. The possible error is greater for one face which has two pairs of strips used in different configurations. The mean of the two values of the thickness for the two pairs is the correct value to use in deriving the value of gravity, but the thickness that is actually measured by comparison with the silica interferometer depends on the relative intensities of the light transmitted by the two strips in the interferometer arrangement and is not well defined. This thickness must be considered to be uncertain by about 0·05 μm.

The coefficient of expansion of fused silica is $0\cdot5 \times 10^{-6}$/degC; the mean thickness of the two blocks at 22 °C (the mean temperature of gravity measurements) is therefore 57106·227 μm with a standard deviation, including the uncertainty of the fused silica interferometer, of 0·021 μm on 4 degrees of freedom.

4. The white light interferometer

(a) *Principles of the system*

The separation of the upper and lower slit pairs is about 1 m and that of the inner faces of the slit blocks about 94·2 cm, a distance that is too great to measure directly in monochromatic light from ordinary sources. It could now be measured directly with light from a gas laser but since that was not available when the apparatus was devised, the method adopted was to use a system of fringes in white light to compare the separation of the blocks with the optical length of a Fabry–Perot interferometer having a spacing that was almost exactly one-fifth of the separation of the blocks and that could be measured directly in terms of wavelengths of monochromatic radiation. The observations of the white light fringes were made photoelectrically and the necessary small changes in the optical length of the Fabry–Perot interferometer were made by altering the pressure of the air in the vessel in which it was placed. This is believed to be the first occasion on which such methods have been applied to the measurements of lengths with fringes in white light, although the fringes themselves have been used for years past with visual observation.

To permit the blocks to be adjusted and to ensure that they should remain in adjustment over many days, they were supported on levelling screws working in invar plates which were fixed to a frame of three vertical rods of invar, the temperature of which was measured with five thermocouples.

The optical system of the white light interferometer is shown in figure 9; the principles on which it is based have been described by Cook & Richardson (1959). If the optical distance between the opposite faces of the two slit blocks is five times that between the plates of the 20 cm Fabry–Perot interferometer, and if the optical train is adjusted so that light reflected between the slit blocks is parallel to light reflected within the interferometer, then light which has suffered one reflexion between the blocks will be in phase with light which has suffered five reflexions within the interferometer, and fringes of interference in white light will be observed in the focal plane of a telescope focused on infinity. A mirror between the slit blocks and the interferometer allowed the latter to be mounted horizontally and controlled the angle between the vector path differences. The fringe intensity is observed photoelectrically in the direction defined by a pinhole placed in the focal plane of the telescope, the system being adjusted so that this direction is normal to the plates of the 20 cm interferometer. The optical paths traversed by the two beams are represented by vectors normal to the reflecting surfaces of the slit blocks on the one hand and of the interferometer on the other. Let the optical retardations in the two beams be $\mathbf{D}_1$ and $\mathbf{D}_2$, where $\mathbf{D}_1$ is $10\,l$, l being the optical length of the 20 cm etalon, and $\mathbf{D}_2$ is $2L$, L being the separation of the slit blocks. The vector difference of $\mathbf{D}_1$ and $\mathbf{D}_2$ is $\boldsymbol{\delta}$ and a maximum intensity in any wavelength occurs when light passes through the system in a direction such that the projection, $\mathbf{p}$, of $\boldsymbol{\delta}$ on that direction is an integral number of wavelengths. The central white fringe occurs in the direction for which $\mathbf{p}$ is zero. In the present arrangement, the direction is fixed by the pinhole to be parallel to $\mathbf{D}_1$ and the optical length of the 20 cm interferometer is varied until the maximum intensity is observed in this direction; $\boldsymbol{\delta}$ is then at right angles to $\mathbf{D}_1$. When the central fringe is on the pinhole, the retardation D_2 is equal to

$$D_1/\cos\theta.$$

The adjustment of the angle θ is important, for as well as introducing a cosine error in the estimation of D_2, it affects the number of fringes in the field of view, that is to say the fringe fraction corresponding to the diameter of the pinhole and therefore the contrast of the fringes as observed photoelectrically. The angular spacing ϕ of the fringes in the field of view follows from the fact that successive projections of $\boldsymbol{\delta}$ on the direction of the pinhole differ by one wavelength; thus

$$\phi = \lambda/\delta.$$

But δ is approximately $D\theta$ and so

$$\phi = \lambda/D\theta.$$

In practice the fringe spacing must exceed 1 mm at a focal length of 40 cm, that is ϕ must be greater than 1/400 for the photoelectric signals to have adequate contrast; taking λ to 0·5 μm and D to be 2×10^6 μm, the maximum value of θ is 1×10^{-4} and the maximum error in D_2 will be $0{\cdot}5\times10^{-8}$. The value of ϕ was however usually two or three times as great as the minimum usable value and it is therefore clear that, provided the fringes can be detected photoelectrically, the measured value of D_2 is not significantly in error.

So far as the white light fringes are concerned, the pinhole may be of any diameter provided that the fringes can be spaced widely enough to give good contrast and, as may be seen from the argument above, the larger the pinhole the better, since it will ensure a large fringe spacing: it will also admit more light, but this is of no practical importance. On the other hand, the adjustment of the mirror to give widely spaced fringes was very delicate and so it was easier to use a small pinhole provided it met the conditions set out above. More stringent conditions are imposed by the observation of the Fabry–Perot fringes formed in monochromatic light in the 20 cm interferometer. With a focal length of 40 cm and a wavelength of 0·5 μm, the diameter of the pinhole should not exceed 1 mm if significant phase errors are to be avoided. Two pinholes have been used, one of 0·5 mm and the other of 0·2 mm diameter and it has been confirmed experimentally that there was no difference between the settings on the fringe maxima so that it would be satisfactory to use either of them. Most of the observations were made with the smaller pinhole.

On account of the dispersion of air, it is not possible to obtain fringes in white light if the space between the blocks is evacuated and the refractive index of the air in this space must be estimated from the measured pressure, temperature and humidity; the temperature was found from three thermocouples placed in the air between the blocks and not in contact with the frame supporting the blocks.

(*b*) *The* 20 *cm interferometer*

The 20 cm interferometer consisted of two optical flats with semi-transparent aluminium coatings in optical contact with the plane parallel ends of an invar tube. The tube was provided with two collars near the ends which could be pulled together with steel screws

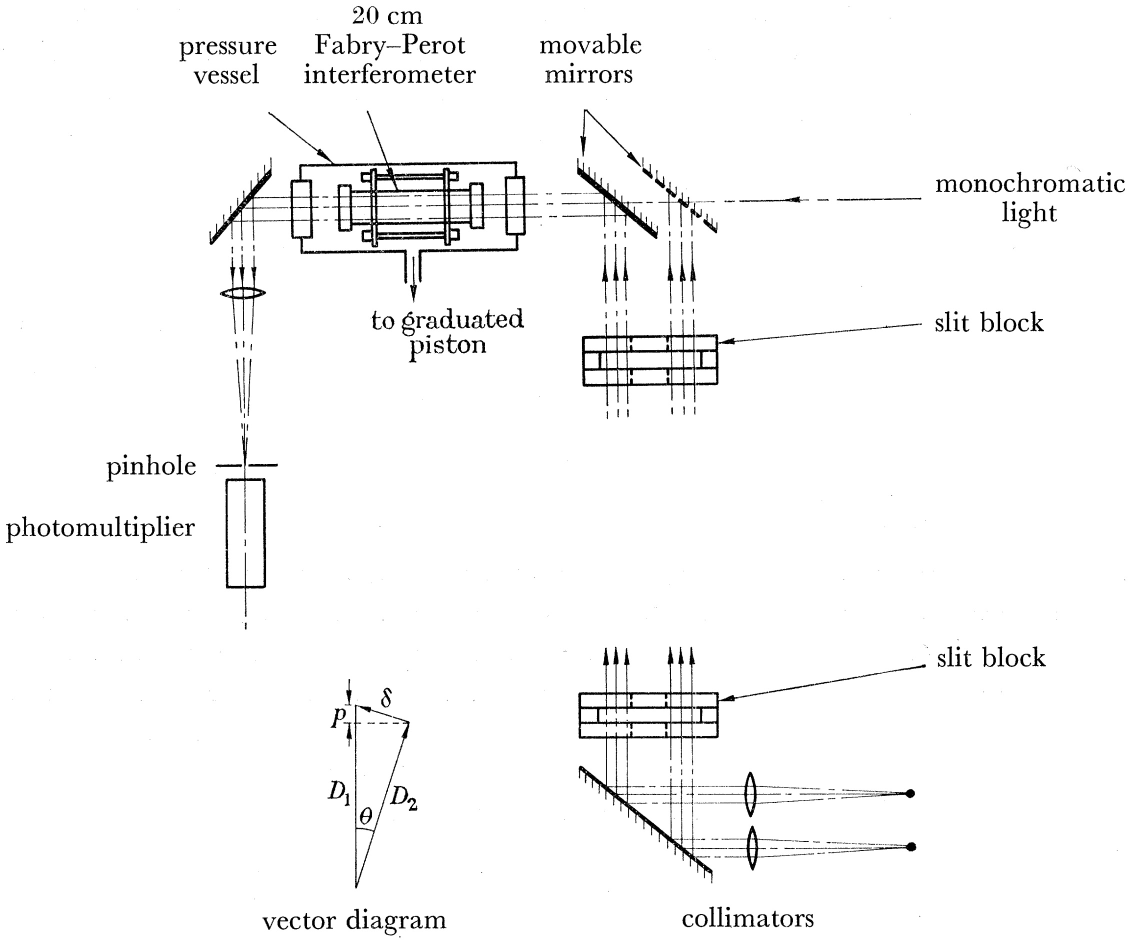

FIGURE 9. Optical system of white light interferometer.

(see Sears & Barrell 1932) to adjust the separation and parallelism of the plates. The interferometer was placed in an airtight cylinder connected to a graduated piston and cylinder with which the pressure of air in the interferometer was altered. The separation of the slit blocks was always adjusted so that white light fringes were obtained when the pressure in the 20 cm interferometer was close to atmospheric.

A mercury-198 lamp illuminated the 20 cm interferometer through a collimator and a rotating sector disk and the fringes were formed in the focal plane of the same telescope with which the fringes in white light were viewed (figure 9): the intensity at the centre of the fringe pattern was measured with the same photomultiplier through the same pinhole as for the white light fringes. The photomultiplier current from the white light was great enough to be observed with a galvanometer of moderate sensitivity, but the current from the monochromatic fringes had to be amplified by an amplifier tuned to the frequency of the rotating sector disk and rectified by a detector synchronized with the disk. In this way, fringes could be observed in the green (λ 5461 Å) and violet (λ 4358 Å) lines of ^{198}Hg, but not reliably in the two yellow lines ($\lambda\lambda$ 5792, 5771 Å) owing to the low visibility of the fringes combined with the low intensity transmitted by the small pinhole and with the need to keep the time constant of the detector short to facilitate visual setting on the fringes. Thus it was not possible to determine the order of interference *ab initio* at each observation because fringes could not be observed in sufficient wavelengths to enable the order to be found unambiguously from the coincidences of the fractional orders of interference. The integral order had therefore to be calculated from an independent determination of the length of the interferometer together with the estimated refractive index of the air inside it, and the observed fringe fractions in the green and violet lines of mercury, which provided a check of consistency, were added to the integral parts of the calculated orders. This turned out to be quite a reliable procedure and gave rise to no uncertainty, because the length of the 20 cm interferometer was very stable.

The definitive measurements of the length of the 20 cm interferometer were made with a krypton-86 lamp, the fringes being photographed through a spectrograph. With such a long interferometer, a number of precautions were necessary to obtain good results. It is desirable to use a large number of wavelengths so that the calculated length shall be unambiguous, and some rather faint lines had to be used. Since the fringe contrast and thus the maximum intensities are in any case low at this path difference, long exposures were needed and the estimation of the refractive index of the air needed care. The observations were made in a room with a controlled temperature and the interferometer was kept in the same enclosure as that in which it was placed in the gravity apparatus since it could thereby be isolated from changes in atmospheric pressure. The container was opened to the atmosphere at the start of an exposure and the temperature and humidity were measured and then it was shut off for the duration of the exposure. The fringes obtained in this way were distinctly sharper than when the interferometer was exposed to the varying atmospheric pressure. Finally, at this path difference, the krypton lamp must be run at the internationally agreed conditions, in a bath of nitrogen kept at the triple point.

Table 3. Length of 20 cm interferometer

(Measurements with krypton-86)

date	number of wavelengths	mean temperature (°C)	length at 20 °C (μm)
Aug. 1964	7	19·33	188529·832 ± 0·009
June 1965	17	21·05	188529·839 ± 0·009

The uncertainties are the standard deviations of the mean values.
Adopted mean value: 188529·836 ± 0·004.

The results are summarized in table 3. The scatter of the separate observations is quite small and the difference between the two sets is not significant. The good agreement between results obtained at different temperatures is consistent with the measured coefficient of expansion of the interferometer.

The coefficient of expansion was found from visual interferometric observations over the range 20 to 25 °C. The result is $1{\cdot}309 \times 10^{-6}$/degC, s.d. 0·011.

(c) *Adjustment and use of the white light interferometer*

The following adjustments must be made to the interferometer to obtain white light fringes in the correct position. The slit blocks must be horizontal, but this is less critical than the parallelism since it concerns only the cosine error in the measured acceleration, whereas the parallelism must be much more carefully adjusted to obtain white light fringes of good contrast. It was therefore sufficient to check the inclination of the bottom block with a sensitive spirit level from time to time; it was always kept within 10″ of the horizontal and usually within 5″. The upper block must be adjusted to be parallel to the lower block and the distance between them must be made very closely equal to five times the length of the 20 cm interferometer.

With the 20 cm interferometer removed, the telescope was adjusted until the lower block imaged the pinhole back on itself and then the upper block was similarly adjusted. The 20 cm interferometer was replaced and adjusted until it also imaged the pinhole on itself. Thereafter, the 20 cm interferometer was not touched because it was in correct alinement for observing the monochromatic fringes, but the spacing of the white light fringes could be altered by tilting the mirror between the blocks and the 20 cm interferometer. At this stage white light fringes would be seen in an eyepiece replacing the pinhole if the separation of the slit blocks was correct or within the range allowed by the possible pressure variation. If not, it was usually possible to find fringes in the light from a high pressure mercury lamp. When fringes had been obtained, the height of the upper slit block was adjusted until the fringes were seen with the pressure in the 20 cm interferometer very close to atmospheric, and at the same time the inclination of the block was re-adjusted to obtain the sharpest fringes. Finally the mirror was tilted to give the greatest possible spacing of the fringes. The slit blocks were then parallel and horizontal and at a separation equal to five times the 20 cm interferometer length to within about 5 parts in 10^7, the white fringes were widely spaced and the centre of the monochromatic fringe system formed in the 20 cm interferometer coincided with the pinhole through which the fringe intensities were measured. On turning the blocks into a different configuration, the fringes were usually seen with good contrast without further adjustment but it might sometimes be necessary to adjust the inclination of the upper block to improve the contrast. For the most part, however, the blocks could be replaced on the levelling screws with inappreciable changes of inclination, a feature of the mechanical design of the apparatus that was invaluable in simplifying the measurements.

An example of the measurements is shown in table 4.

Using the five thermocouples on the invar frame, three in the air between the blocks and one on the interferometer container, the temperatures of the frame, air and interferometer were read. The pressure and humidity of the air were measured with the interferometer container open and the container was then closed when the graduated piston was at a standard setting (3·500). The white light fringes were observed and the piston settings for

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TABLE 4. EXAMPLE OF WHITE LIGHT FRINGE OBSERVATIONS

thermo-couple	position	initial (°C)	initial mean (°C)	final (°C)	final mean (°C)
1	frame	21·59	22·05	21·61	22·04
2		21·97		22·00	
3		22·03		22·03	
4		22·26		22·24	
5		22·38		22·34	
6	air	22·04	22·08	22·03	22·09
7		21·82		21·89	
8		22·38		22·35	
9	interferometer	22·58		22·51	

interferometer closed at piston setting 3·500.
pressure 1016·19 mb, water vapour pressure 10 mb.

settings on white fringes, piston readings at maxima:

	side A 1st	side A 2nd	side B 1st	side B 2nd
	3·633	3·641	3·607	3·618
	3·634	3·641	3·607	3·619
mean	3·637		3·613	

settings on monochromatic fringes

green (5461 Å)	3·402	3·528	3·656
	—	3·525	3·652
	3·402	3·523	—
mean	3·402	3·525	3·654
violet (4358 Å)	3·441	3·548	3·648
	3·444	3·545	—
mean	3·442	3·546	3·648

final reading of pressure 1016·47 mb water vapour pressure 10 mb

mean conditions

temperature:	frame	22·04 °C
	air	22·08
	interferometer	22·53
pressure		1016·33 mb
water vapour pressure		10 mb

	green	violet
mean refractivity $[10^6(n-1)]$		
in 20 cm interferometer at 3·500	271·165	274·277
between blocks	271·656	274·774
calculated order in 20 cm interferometer at 3·500	690487·89	865142·64
observed fraction	0·82	0·57
observed minus calculated	−0·07	−0·07
observed order at white fringes		
side A	690488·91	865143·91
side B	8·71	3·67
separation of blocks (μm at 22·04 °C)		
side A	942653·23	942653·21
side B	2·96	2·95

maximum photomultiplier current were recorded; the optical system was such that the separation of the blocks could be measured on both sides of the central hole and readings were made for both positions (A and B). The 20 cm interferometer was now illuminated by the mercury-198 lamp and the piston settings for maxima of the green and violet fringes were recorded, settings being made on both sides of those for the white light fringes. The temperature of the 20 cm interferometer was read again and settings on the white fringes were repeated, the measurements being concluded with further readings of the air pressure, humidity and the temperatures.

The order of interference in the 20 cm interferometer was calculated from the refractive index and temperature and the fractional part was compared with the observed fraction at the standard piston setting, 3·500. In general, the observed and calculated values are in satisfactory agreement. The mean differences are:

green	−0·044,	s.d. 0·11;
violet	0·00,	s.d. 0·16.

In part, this is because the increase of order due to the expansion of the invar tube is almost exactly compensated by the change of refractive index with temperature so that the calculated order is almost independent of the temperature of the interferometer, a convenient situation when, as often happens, the temperature is changing appreciably.

The order of interference in the 20 cm interferometer at which the white light fringe maximum occurs is equal to the integral part as calculated plus the observed fraction corresponding to the difference between the piston readings at which a monochromatic fringe and the white maximum occurred; the order between the slit blocks is exactly five times the order so found.

Finally, the mechanical separation of the slit blocks was calculated from this order and the refractive index of the air, a value being obtained from each of the two wavelengths, and the lengths were reduced to the standard temperature of 22 °C using the coefficient of expansion given below.

The condition for the central white fringe is

$$\bar{\mu}_1 t_1 = 5\bar{\mu}_2 t_2,$$

where t_1 is the separation of the blocks, t_2 is the length of the 20 cm interferometer, $\bar{\mu}_1$ is the refractive index of the air between the blocks, $\bar{\mu}_2$ is that of the air in the interferometer, and the bar ($\bar{\mu}$) denotes an average over the wavelength response (lamp plus photomultiplier) of the white light system.

If n_2 is the measured order of the 20 cm interferometer in monochromatic light.

$$n_2\lambda = \mu_2 t_2,$$

where λ is the vacuum wavelength of the monochromatic light and μ_2 is the refractive index for this wavelength at the same air density as that for the white light measurements.

Hence

$$t_1 = 5\frac{\bar{\mu}_2}{\mu_2}\frac{n_2\lambda}{\bar{\mu}_1}.$$

The equation used to calculate the separation of the blocks is

$$t_1 = \frac{5n_2\lambda}{\mu_1},$$

μ_1 being the refractive index for the monochromatic light at the density of the air between the blocks. An error is thus committed of relative magnitude

$$\frac{1}{\mu_1} - \frac{\bar{\mu}_2}{\mu_2}\frac{1}{\bar{\mu}_1}.$$

Now let the refractive index be written as

$$\mu = 1 + dl,$$

where d is proportional to the air density and l, which is very nearly 1, depends on the wavelength.

Then, with an obvious notation,

$$\frac{1}{\mu_1} - \frac{\bar{\mu}_2}{\mu_2}\frac{1}{\bar{\mu}_1} = \frac{1}{1+d_1 l} - \frac{1+d_2\bar{l}}{1+d_2 l}\frac{1}{1+d_1\bar{l}}$$
$$= (d_1 - d_2)(\bar{l} - l)/\mu_1\mu_2\bar{\mu}_1.$$

The wavelength factor varies by about 1% between the green and the violet and so, since the average wavelength of the white light system is near the green, $(\bar{l} - l)$ will be about 1% for the violet and much smaller for the green. In practice the difference between the density factors for the blocks and the interferometer never exceeded 1 in 10^6 and accordingly the error in using the approximate equation to calculate the separation of the blocks was less than 1 in 10^8 for the violet and even less for the green.

Complete measurements as just described were generally made each time the blocks were turned to a new configuration and in addition the separation was checked after a series of timing measurements by white light fringe measurements without the monochromatic fringe observations. Because of the high stability of the 20 cm interferometer this leads to no significant loss in accuracy and saves appreciable time. There are a few sets of measurements for which no monochromatic observations were made on either occasion.

The coefficient of expansion of the invar frame was obtained from measurements made at 21 and 25 °C. The mean value is

$$2{\cdot}2\ \mu\text{m/degC}, \quad \text{s.d. } 0{\cdot}05.$$

The uncertainty is rather large, possibly because of real differences between the invar rods, but the difference between the temperature at which the separation of the blocks was measured and that at which they were used in timing the flight of the ball was always less than 1 degC and usually less than 0·5 degC so that the uncertainty in g arising from the uncertainty of the coefficient is less than $0{\cdot}5 \times 10^{-7}$ (see also §11).

The separation of the blocks was measured at atmospheric pressure but the flight of the ball was timed *in vacuo*; the effective separation is thus greater than the measured separation because of the compression of the invar rods under the hydrostatic pressure of the air and a correction must be applied to the apparent value of g (§11(a)).

5. The timing system

Because the initial velocity of the ball varied by 1 or 2%, the times at the upper and lower slit blocks could not be predicted more closely although the value of gravity is already known to a few parts in a million. The entire time intervals had therefore to be measured and not just the residuals from some calculated values; the timing system was therefore based on counters that counted the number of cycles of a 1 MHz signal between the two pulses at one or other of the slit pairs. Counters by themselves, would not give sufficient accuracy, although with a 10 MHz signal the resolution would be adequate, because the form and amplitude of the pulse from the downward passage of the ball may differ from that from the upper passage and therefore the counter might not trigger at corresponding instants. The pulses were therefore recorded photographically and the final time measurements were made from the traces.

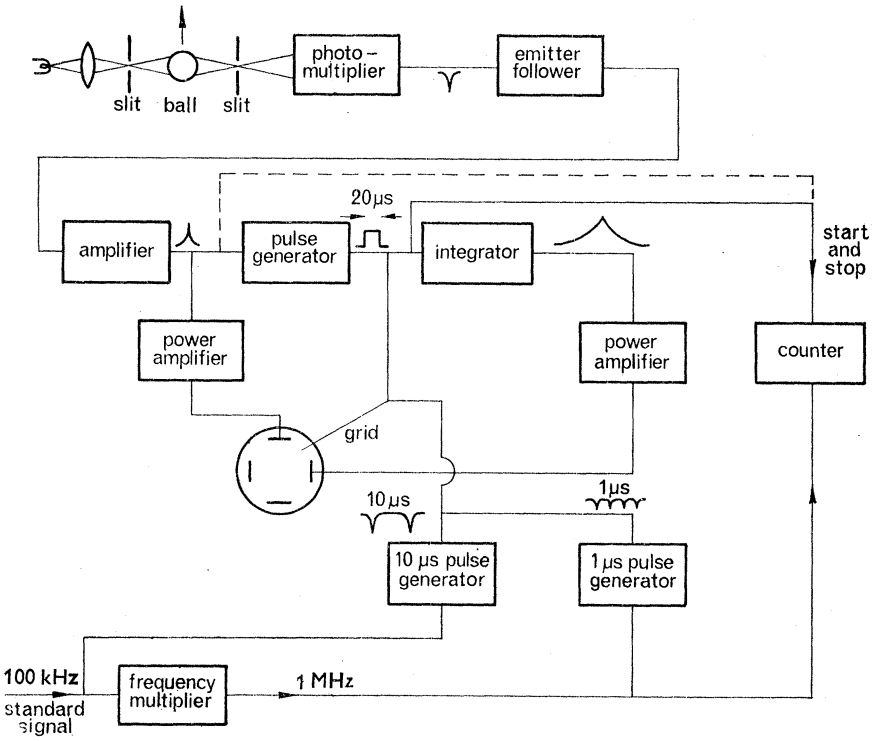

FIGURE 10. Timing system. There are two similar systems, one for each slit block.

The timing system is indicated in figure 10. The amplified pulse from the photomultiplier triggered a pulse generator that produced a pulse lasting for about 20 μs. This was integrated to provide a time base sweep for a cathode-ray oscilloscope to the vertical deflexion of which the photomultiplier pulse was applied. At the same time the 20 μs pulse was applied to the grid of the tube to brighten the trace during the single sweep, so enabling a photograph of the trace to be taken on a plate continuously exposed to the oscilloscope. Pulses derived from the 1 MHz signal applied to the counters were used to brighten the trace every microsecond, together with larger pulses every 10 μs. The trace was shifted vertically after the first pulse so that the second might be recorded without confusion. The counters were started and stopped not from the pulses from the photomultiplier but from the leading edges of the 20 μs pulses. The timing depended on the 1 MHz signal derived from the standard 100 kHz signal of the N.P.L.

Three voltage levels were crucial in the operation of the system. The pulse amplitude at which the 20 μs pulse generator was triggered could be varied and the amplitude of the pulse from the photomultiplier could be altered by changing the voltage applied to the photomultiplier, and these two levels were adjusted to give the most reliable triggering. The noise pulses from stray light are the determining factor and it was found best to set the trigger level and the pulse amplitude as high as possible so that the difference in amplitude between the noise pulses and the pulse from the ball was as great as possible. On the other hand, an upper limit was set by saturation of the pulse from the ball which occurred first in an emitter follower that followed the photomultiplier. A third voltage level determined the triggering of the counters. The 20 μs pulses were applied to a bistable circuit and it was possible for this to switch on at a different level from that at which it switched off, a difficulty that was largely overcome by differentiating the 20 μs pulse so that the pulse applied to the bistable circuit lasted for a much shorter time; even so care had to be taken to set the amplitude of this pulse correctly. To some extent, errors in the starting and stopping of the counter were not important since the exact time should be obtained from the photographs of the cathode-ray tube traces but it was desirable that the counters should be correct if possible; moreover, some work was done with the counters alone, triggered either from the 20 μs pulses or from the photomultiplier pulses directly.

Many preliminary measurements and some of the observations included in the final value of gravity were made with the counters alone. In such observations there are additional systematic as well as random errors, but since it was considerably faster to use the counters by themselves, it was worth while to do so if the systematic errors were not important, for example in experiments on the effect of air resistance on the measured acceleration, for in those experiments it is the change in ΔT^2 with air pressure that was required and not the actual value of the acceleration. Some observations with the counters alone have also been included in the final value of gravity but only when the systematic error was calibrated with photographic observations. The details of such observations will be given in §10.

6. Catapult and vacuum vessel

(a) *Catapult* (figure 11)

The catapult for shooting the ball up is similar to the cross-bow. Before firing, the ball rests in a cone at the top of a vertical rod provided with two semicircular bearing surfaces, one at each end of the rod, which work in a fixed vertical V-guide so that the rod is constrained to move vertically. The guide is fixed to a frame which can be adjusted by means of levelling screws so that the flight of the ball is as nearly as may be vertical. To minimize friction in a good vacuum, the bearing surfaces on the rod are sintered bronze impregnated with polytetrafluoroethylene and the surfaces of the V-guide are ground to a high finish on nitrided steel. The rod is held against the V-guide throughout its motion by two steel springs attached to the frame carrying the guide and is propelled along the guide by an elastic cord fixed at its mid-point to the middle of the rod and at its two ends to the frame. The tension in the cord is adjustable to vary the height to which the ball is thrown. The rod is taken to the bottom of its travel by a setting mechanism and is held there by an electromagnet until it is to be released.

After a shot, the ball was caught and replaced on the cone and the rod was re-set ready for another shot, all without letting air into the apparatus. To catch the ball, a split cup

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was provided above the rod and below the lower slit block. Before the ball was fired the two sections of the cup were held apart to allow the ball to pass between them, but on firing, a catch was released by a relay and the two halves of the cup came together, the ball falling into them on its return. The cup could then be opened again to drop the ball onto the cone at the top of the rod. Thus it was possible for the ball to be fired at intervals of no more than 2 or 3 min.

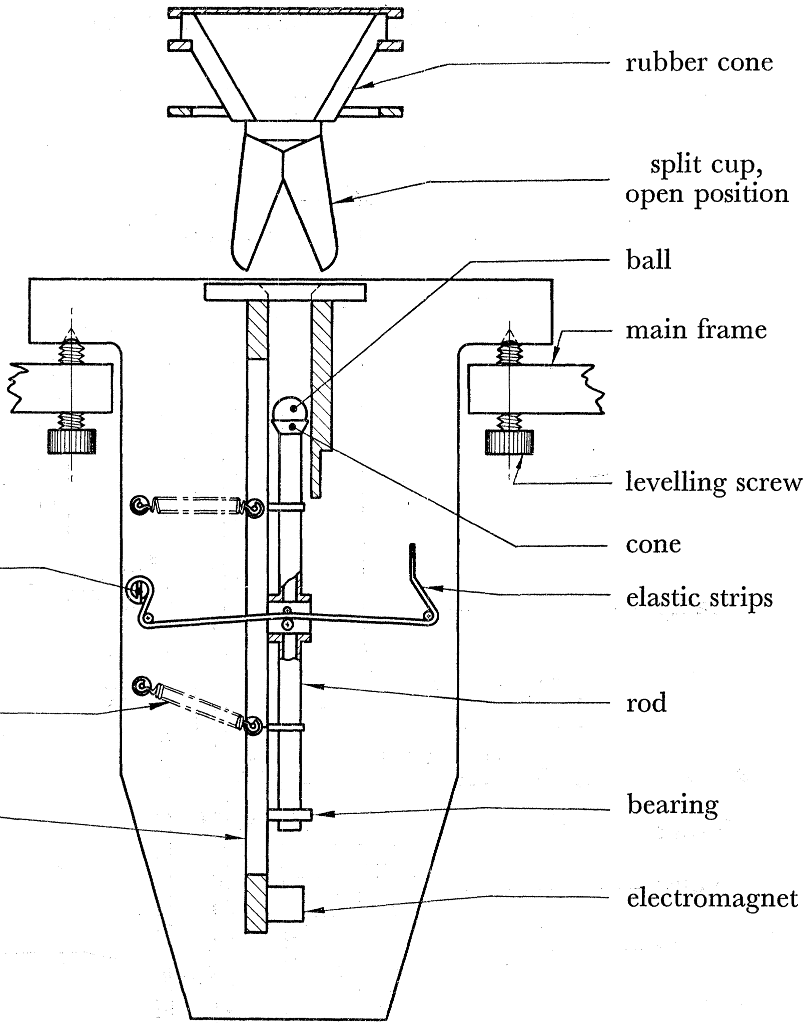

FIGURE 11. Catapult.

Great care was taken in the design of the catapult to keep any vertical impulse given to the rest of the apparatus very small, for any such impulse, occurring always at the same time relative to the flight of the ball, would give rise to a systematic error in the measured acceleration, unlike the effects of microseismic movements which are random with respect to the motion of the ball. The behaviour of the catapult was therefore carefully checked with a vertical seismometer and it was found that any vertical movement that could be attributed to the release of the rod was less than 0·1 μm, but that the release of the split cup did produce a detectable effect, of not more than 0·1 μm, in the form of a damped train of oscillations of frequency about 1 kHz, lasting for a few hundredths of a second. A delay was therefore introduced electrically between the firing of the ball and the closing of the cup, so that the cup was closed when the ball was midway between the lower and upper blocks and the small vibration produced by the cup release had no effect on the measured times.

(*b*) *Horizontal monitor*

Since the pulses from the slit system are very sensitive to the horizontal position of the ball in the plane of the slits, a means for measuring that position was provided. Two parallel beams of light were set up by collimators, the widths and positions of the beams being defined

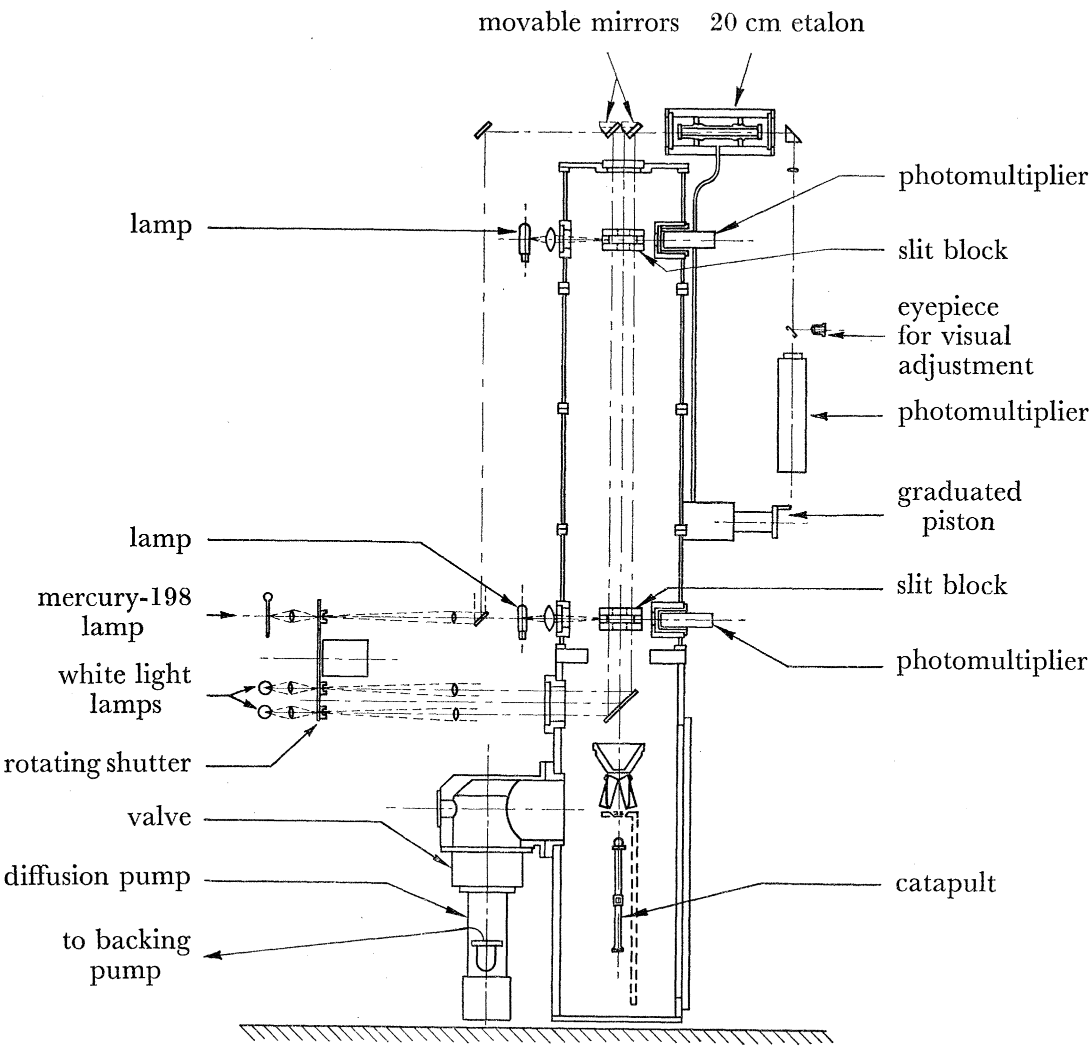

FIGURE 12. General diagram of vacuum vessel and optical apparatus.

by slits so that when the ball was central it intercepted half of each beam. The beams fell one on each of two photoresistive cells in a bridge network so that if the ball was off centre, the bridge was out of balance; the out-of-balance signal of about 1 ms duration was displayed on an oscilloscope. Thus any horizontal displacement of the ball at right angles to the beams of light could be detected and estimated approximately. Two systems were provided to detect displacements in the two horizontal directions. The device was sensitive to about 0·01 mm and enabled adjustments to be made to the levels of the catapult frame while maintaining a vacuum in the vessel.

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(*c*) *The vacuum vessel* (figure 12)

The vacuum vessel comprises two distinct chambers, one containing the catapult and the other the slit blocks on their invar frame. The lower, catapult, chamber is made in one piece, with a door for putting the catapult in position. The upper surface forms part of a hollow square section beam which is the main support for the whole apparatus and which rests on three heavy levelling screws; to it is fixed the invar frame carrying the slit blocks and the upper, cylindrical part of the vacuum vessel, which is otherwise quite independent of the invar frame and slit blocks.

The vessel was pumped continuously throughout timing observations and it was therefore necessary to check with the seismometer that the vacuum pumps, especially the rotary backing pump, did not cause vertical movements of the apparatus. The rotary pump was placed on anti-vibration mounts to avoid transmission of vibration through the floor of the laboratory and a flexible section of pipe was inserted between it and the diffusion pump. These precautions were sufficient, for although air-borne vibration was observed when the air pressure in the backing line was still high, it disappeared at low pressures, and under working conditions there was no detectable vertical movement.

The seismometer (§9), fixed to the surface which carried the invar frame, measured the vertical movements of the frame.

7. Electrical charging of the glass ball

The most serious difficulty in the conduct of the determination has been the electrostatic charging of the glass ball. Although satisfactory preliminary observations were made in air at pressures of 45 μb, the behaviour of an ordinary glass ball became very erratic when the pressure was reduced below about 15 μb, the measured acceleration often varied greatly and such large horizontal displacements were given to the ball that sometimes it did not return through the hole in the lower slit block. At first it was thought that a large increase of friction of the catapult rod on the guides was responsible but it was shown not to be significant and electrostatic charging of the ball was examined; rather simple apparatus sufficed to detect substantial charges.

The charge on the ball was detected by the voltage induced on an electrode connected to a cathode-ray oscilloscope. The electrode was a vertical tube through which the ball passed; the diameter of the tube was about twice that of the ball and the length about three times. When the ball was at the centre of the tube, a charge nearly equal to that on the ball was induced on the tube and with the tube connected to a capacitor of suitable value, the voltage appearing on the capacitor and observed on the oscilloscope was proportional to the charge induced on the tube and had the same variation in time. (The capacitor must be large enough to preserve the shape of the charge pulse and yet small enough to give a good voltage sensitivity.) Quite large charges were detected, the equivalent voltages on the ball being of the order of 1 to 5 kV, assuming the capacity of the ball to be equal to its radius.

The charge develops when the ball separates from the cone on the catapult rod and changes very little in the course of the flight, provided the air pressure is not close to atmospheric. Charging is practically never observed at pressures of 30 μb or greater but with a plain glass ball charging sets in at between 7 and 15 μb and rapidly attains its maximum value as the pressure is reduced. The material of the cone (conducting or non-conducting) has little effect on the charging of a plain glass ball, nor does the ionization of the residual air in the region of the contact of the ball and cone. It is only the conductivity of the surface of the ball which has any major effect and it was found that by coating the ball with a layer of surface resistivity of about $10^5\,\Omega/\text{cm}^2$, the charging could be practically eliminated. (A metal ball never shows any charging.) The ball was therefore coated with a layer of indium oxide which, for such a resistivity, reduced the overall optical transmission of the ball by about one half. Unfortunately, these coatings are not wholly reproducible and the resistivity increases with time, so that all the definitive measurements of gravity were made with a ball that had a somewhat higher resistivity than was desirable and which developed some charge at pressures below $1{\cdot}5\,\mu$b. With this ball it was necessary to use an earthed metal cone on the catapult rod.

Because of the risk of charging at the lowest pressures, the definitive measurements were made at pressures of the order of $15\,\mu$b, at which charging is always negligible. It is then necessary to be very sure that the measured acceleration is independent of air resistance at such a pressure and this question was investigated experimentally as described in the next section. It is important that the resistivity of the coatings enabled the pressure in those experiments to be taken down to $0{\cdot}03\,\mu$b.

8. The effect of air resistance

It has been shown (Cook 1965*a*) that the effect of air resistance on the measured acceleration is zero to first order provided only that the resistance is proportional to the velocity of the ball. At the pressures involved, it is to be expected that that would be so but it is important to confirm the prediction experimentally, especially since it is desirable, in order to avoid the charging difficulties, to work at pressures at which the direct air resistance on a falling body is many parts in a million.

Three experiments have been carried out to check the effect of air resistance. In each of them the measurements were made differentially, that is to say, the distance between the slit blocks was not measured but the blocks were left always in the same position throughout the experiment and a correction was applied for the changing temperature of the invar frame and, when glass slit blocks were used, for any difference between the temperatures of the blocks. The results of the three experiments are given in table 5. The uncertainties of the observations of time are much the same in each determination, the greater scatter of the counter timing being compensated by taking more observations. Systematic errors in the timing have no effect on the result. The first experiment was made with the original glass slit blocks and covered the greatest range of pressure but extended over five days. The scatter of the observations is however quite satisfactory. In the second set, counters only were used in the timing and it was possible to complete the measurements within a day. The range of pressure is, however, much less. The third set of measurements was made in order to have some observations with the ball actually used for the definitive measurements of gravity with which, as it had a larger resistivity, it was not possible to work at the lowest pressures, although some measurements were secured at $1{\cdot}5\,\mu$b. The temperature was changing rapidly during these observations and the gradient of temperature in the invar frame also changed, far more than had ever been found before, and the measured times showed a systematic dependence on the gradient in the frame.

The uncertainties of the pressure coefficients are effectively inversely proportional to the ranges of pressure in the three determinations and the weighted mean value, which really depends only on the first determination, is equivalent to $1{\cdot}1 \times 10^{-3}$ mgal/μb. Now the static buoyancy term for a glass ball with a density of about 2 g/cm^3 is $0{\cdot}6 \times 10^{-3}$ mgal/μb and this should be doubled to allow for the inertia of the air moving with the ball. Thus if the resistance were strictly zero, the expected effect would be about $1{\cdot}2 \times 10^{-3}$ mgal/μb. The observed value is so close that it can be said that at pressures as high as 1·2 mb, the air resistance is proportional to velocity.

Table 5. Measurements of effect of air resistance

Data fitted to equation of form:

$$\Delta T^2 = \Delta T_0^2 + ap$$

units of ΔT_0^2, 10^{-6} s^2

of a, 10^{-9} s^2/μb

date	period of experiment (days)	timing	range of pressure (μb)	pressure coefficient a	standard deviations of ΔT_0^2	standard deviations of a
Jan. 1963	5	photographic	0·04–1200	+1·0	0·74	1·3
June 1963	1	counters only	0·04–75	+5·1	0·2	16
May 1965	1	counters only	1·4–56	−3·0	0·07	1·2

A term $b\delta\theta$ was also fitted to the results of May 1965. $\delta\theta$ is the difference between the temperatures of the upper and lower slit blocks

$$b = -0{\cdot}92 \times 10^{-6}\ \mathrm{s^2/degC} \text{ with a standard deviation of } 0{\cdot}80.$$

These experiments also show that in the conditions in which they were done, there is no detectable electrostatic charging, for the charge varies discontinuously with pressure and therefore the vertical accelerations that it would produce would be distinguishable from changes in acceleration due to air resistance. This conclusion applies in particular to the conditions of the definitive measurements of gravity.

9. Microseisms

All free motion determinations of gravity are affected by movements of the apparatus because the quantity that is measured is some sort of mean acceleration of the falling body relative to the apparatus, whereas the acceleration that is required is relative to the centre of mass of the earth. The consequential errors depend on the period of the movements and on the phase relative to the movement of the falling body, and there are two principal classes of movement, those produced within the apparatus and having a systematic relation to the movement of the falling body, and those originating outside the apparatus and having a random relation to the falling body. The former are much the more serious because, unlike the latter, the effects cannot be reduced simply by taking a large enough number of observations. The systematic motions of the apparatus usually consist of a train of damped vibrations of a well defined period excited by the operation of some part of the apparatus. In the present apparatus, all parts of which are mechanically very stiff, the dominant period is of the order of 1 ms and, as was explained in §6, care was taken to keep the amplitude of these oscillations very small and to arrange that they occurred at a time when they had no effect on the measured acceleration. Three sources of random movement are effective in the present experiment, vibrations of well defined period but of negligible amplitude from the rotary vacuum pump, locally generated microseisms of short period and microseisms of long period generated at a distance. The pump vibrations were kept small as described in §6, but nothing can be done to reduce the true microseisms. Microseisms increase the scatter of the observed values of gravity but do not introduce any systematic error, as has been discussed in detail (Cook 1965*a*) for representative spectra of random movements. At the N.P.L. there are two distinct spectra, the long period one, which arises from storms in the Atlantic and has a peak amplitude of about 0·5 μm at a period of about 5 s (with some seasonal variation) and the short period one which is generated locally by machinery and traffic, and has a peak amplitude of about 0·1 μm at a period of about 0·2 s. The largest effect on the gravity measurements would be produced by microseisms with a period equal to the time interval above the lower slit plane, that is about 0·95 s, and it is fortunate that the microseismic energy at this period is very small at the N.P.L.

The apparatus vibrations and the microseisms have been recorded with a long period (7 s) seismometer that measures the vertical movement of the apparatus, in fact the surface of the vacuum vessel on which the invar frame carrying the slit blocks is assembled.

In the conditions of the experiment, the expected standard deviation of a single value of ΔT^2 due to microseisms is 0·75 p/M; the dominant contribution to the scatter is from the long period microseisms, the short period ones of local origin having a negligible effect.

10. Measurements of gravity

(*a*) *Procedure*

As explained in §2, four sets of length and time measurements made with the slit blocks in the four possible configurations constitute a complete set of observations from which it is possible to calculate a value of gravity and the effective positions of the slit planes within the two blocks. For each configuration the separation of the blocks was measured with the apparatus open to the atmosphere, the apparatus was then evacuated and about ten flights of the ball were timed, measurements of the temperature of the invar frame being made at the same time to enable a correction to be applied for the thermal expansion of the frame. Finally air was again let into the apparatus and the separation of the blocks was checked with observations of the white fringes only, omitting settings on the monochromatic fringes.

Such a set of measurements could be completed in half a day if no difficulty occurred. Ten shots timed with the photographic system took about 1 h, allowing for occasions on which the counters did not trigger, but with the counters alone about three times as many shots were timed. A preliminary set of observations was made with the counters alone. In the definitive measurements the photographic system was always used with the lower slits, but in one set the counter alone was used with the upper slit; the photographic system was used on both the upper and lower slits for the other two definitive sets. The numbers of shots timed were such that the standard deviation of the mean time of a set of shots was about the same however they were timed.

The value of gravity has been derived from three complete sets of four measurements in the four configurations. After the first complete set the blocks were turned round so that the slit that had been next to the photomultiplier was now next to the lamp, and after the second set the blocks were interchanged top to bottom.

(*b*) *Results*

The measurements of length and time are given chronologically in tables 6 and 7; all complete time measurements are included. The configurations of the blocks are numbered 1 to 4 and the sets of measurements are labelled *P*, *A*, *B* and *C*. *P* denotes the preliminary set in which counters only were used for timing.

Table 6. Measured separation of the slit blocks

(Lengths in μm at 22·00 °C)

set	con-figuration	full measurement	green–violet	check measurement	check – full	difference between sides
P	2	942652·54	−0·18	{942652·56 2·75	+0·02 +0·21}	0·14
	4	3·49	−0·04	3·28	−0·21	0·14
	3	2·81	−0·04	3·04	+0·23	0·08
	1	3·00	+0·02	2·90	−0·10	0·26
	2†	2·53	0·00	2·86	+0·33	0·07
A	2†	942652·53	0·00	942652·86	+0·33	0·07
	4	3·90	−0·02	4·05	+0·15	0·02
	3	2·92	+0·08	2·81	−0·11	0·08
	2	—	—	2·73	—	0·09
	1	3·01‡	—	3·06	+0·05	0·44
B	3	942652·78‡	—	942653·04	+0·26	0·10§
	2	2·42	−0·04	2·56	+0·14	0·61
	1	2·50	+0·22	2·80	+0·30	0·70
	4	3·29	−0·08	—	—	0·24
C	3	942651·45	+0·03	942652·26	+0·81	0·22
	2	1·53	+0·04	1·62	+0·09	0·31
	1	1·92	−0·08	2·44	+0·52	0·38
	4	2·77	+0·18	2·78	+0·01	0·11
	3	2·20‖	—	—	—	0·14

† These two are one and the same.
‡ No monochromatic observations.
§ Difference between sides changed by 0·16 μm.
‖ Green measurements only.

Measurements were made on the blocks on both sides of the central hole and the difference between the two lengths shows how far the blocks were out of parallel. The largest difference is 0·70 μm; as explained in §2, it will not cause any significant error in the apparent acceleration.

The 'full' measurements of the separation of the blocks are those in which monochromatic measurements were made of the order of interference in the 20 cm etalon and 'check' measurements were those in which the length of the etalon was assumed to be that measured independently. Monochromatic measurements were made in both the green and violet on almost all occasions and the differences between the observed and calculated orders in the 20 cm interferometer are slightly less than expected from the scatter of readings on the fringe settings.

Most of the differences between the full and the check measurements (table 6) are no larger than would be expected from the scatter of results between the green and violet monochromatic observations, but two are greater and it seems that changes in the positions of the blocks have occurred, for apart from one instance, the differences are too large to be accounted for by changes of temperature or pressure or temperature gradient, between the corresponding full and check observations.† The mean of the full and check measurements has always been adopted; the standard deviation of such a mean value derived from the differences between full and check measurements is 0·17 μm.

Table 7. Mean values of measured ΔT^2

($\Delta T^2 = T_a^2 - T_b^2$)

set	configuration	ΔT^2 (10^{-6} s^2)	no. of shots	s.d. of mean
P	2	815102·88	24	0·20
	4	191·74	25	0·14
	3	197·18	25	0·16
	1	096·90	17	0·21
	2	102·88	23	0·21
A	2	815102·86	10	0·27
	4	192·73	9	0·41
	3	197·34	9	0·34
	2	104·26	9	0·32
	1	097·52	10	0·32
B	3	815197·35	10	0·21
	2	102·61	10	0·23
	1	097·76	10	0·28
	4	191·77	16	0·30
C	3	815197·08	—	—
	2	102·89	10	0·24
	1	096·71	13	0·18
	4	191·77	12	0·28
	3	196·73	12	0·24

Mean variance of mean value $0·0884 \times 10^{-12}$ s^4 s.d. $0·30 \times 10^{-6}$ s^2.

In deriving the time measurements given in table 7, corrections have been applied to the observed times when counters only were used. In the definitive measurements of sets *A*, *B* and *C*, photography was always used with the lower blocks and from the measurements in which photography was used with both blocks, the mean error of the counter alone on the upper block was found to be +0·34 μs, entailing a correction of 0·14 μs^2 to $(T_a^2 - T_b^2)$ when photography was not used on the upper block. A correction was also derived for the use of counters only with the bottom block when the counter was triggered from the 20 μs time base pulse, but the preliminary results, *P*, were obtained with the lower counter triggered from the photomultiplier signal directly and the results show that the correction is inapplicable to these conditions; a value of gravity cannot therefore be derived from the *P* group but these data are none the less of use in checking the general consistency of the observations.

The figures given in table 8 suggest that the scatter of the observed times arises mainly from microseismic disturbances. The error of reading the time on the photographs of the fast

† These changes probably arise from disturbance of the lower block when a cover, designed to protect it from a ball in erratic flight, was placed on the block.

pulses is estimated to be about 0·2 μs which corresponds to a variance of $(T_a^2 - T_b^2)$ of 0·13 $(\mu s^2)^2$ so that the variance due to microseismic disturbances is estimated to be

$$(0{\cdot}63 - 0{\cdot}13) = 0{\cdot}50\,(\mu s^2)^2,$$

whereas the value expected from estimates of the spectrum of long period microseisms (§9) is nearly 0·40 $(\mu s^2)^2$. The agreement is quite close.

The standard deviation of the mean of a set of ΔT^2 (usually a set of 10) is 0·30 μs^2.

Table 8. Summary and analysis of variances of time measurements

Variances are of ΔT^2 for single shots in units of 10^{-12} s^4.

1. photography at both levels	0·633
2. counters only at both levels	1·587
3. counter at upper level, photography at lower	0·942
(*a*) upper counter variance, (3 − 1)	0·309
(*b*) lower counter variance, (2 − 3)	0·645
(*c*) upper and lower counter variance, (*a* + *b*)	0·954

The mean values of time and length used to calculate the value of gravity are given in table 9. Let the distance x_a of the slit plane in one of the blocks from one of the horizontal faces of that block be written as

$$\tfrac{1}{2}t_a + \xi_a,$$

where t_a is the thickness of that block. Then in one configuration of the blocks, the effective separation of the two slit planes is

$$D_1 + \xi_a + \xi_b + t_m,$$

where ξ_a and ξ_b are the values for the two blocks and t_m is the mean thickness of the two blocks, equal to 57106·227 μm at 22·00 °C (§3).

Put $$L_1 = D_1 + t_m \quad \text{and} \quad \Delta T_1^2 = (T_{a1}^2 - T_{b1}^2).$$

Then $$L_1 + \xi_a + \xi_b = \tfrac{1}{8}g\,\Delta T_1^2;$$

or writing $$g = g_T + \delta g,$$

we have $$\xi_a + \xi_b - \tau\delta g = \Gamma\Delta T_1^2 - L_1 = z_1 \quad \text{say},$$

where $$\tau = \tfrac{1}{8}\Delta T^2,$$

ΔT^2 being a mean value, and $$\Gamma = \tfrac{1}{8}g_T.$$

It is worth while to make corrections for the tidal variations of gravity.

Let the instantaneous value of gravity, g_i, be

$$g_i = g_0 + g',$$

where g_0 is the value of gravity without tidal perturbation and g' is the tidal effect. Putting

$$g_0 = g_T + \delta g,$$

we have $$L_1 + \xi_a + \xi_b = \tfrac{1}{8}(g_T + g' + \delta g)\,\Delta T_1^2$$

and z_1 becomes $$\Gamma\Delta T_1^2 - L_1 + \tau g'.$$

The tidal corrections (for which I am indebted to Dr W. Bullerwell) were obtained from tables published in *Geophysical prospecting*.

TABLE 9. SUMMARY OF LENGTH AND TIME MEASUREMENTS

set	configuration	ΔT^2 (10^{-6} s^2)	$\Gamma\Delta T^2$ (μm)	D (μm)	$L = D+t_m$ (μm)	tidal correction $\tau g'$ (μm)	z (μm)	$O-C$ (μm)
A	1	815097·52	999696·73	942653·04	999759·27	0·00	−62·54	−0·26
	2	103·56	704·14	2·72	58·95	0·00	−54·81	+0·33
	3	197·34	819·16	2·86	59·09	−0·07	+60·00	−0·22
	4	192·73	813·50	3·98	60·21	+0·06	+53·35	+0·27
B	1	815097·76	999697·02	942652·65	999758·88	+0·02	−61·84	+0·44
	2	102·61	702·97	2·49	8·72	−0·10	−55·85	−0·71
	3	197·35	819·17	2·91	9·14	+0·02	+60·05	−0·17
	4	191·77	812·33	3·29	9·52	+0·06	+52·87	−0·21
C	1	815096·71	999695·74	942652·18	999758·41	0·00	−62·67	−0·39
	2	102·89	703·32	1·58	7·81	−0·04	−54·53	+0·61
	3	196·90	818·62	2·03	8·26	+0·01	+60·37	+0·15
	4	191·77	812·33	2·78	9·01	−0·04	+53·28	+0·20
P	1	815096·90	999695·97	942652·95	999759·18	—	−63·21	—
	2	102·88	703·30	2·65	8·88	—	−55·58	—
	3	197·18	818·96	2·92	9·15	—	+59·81	—
	4	191·74	812·29	3·38	9·61	—	+52·68	—

$g_T = 981180{\cdot}00$ mgal, $\Gamma = \tfrac{1}{8}g_T = 1{\cdot}22647500$ m/s^2, $\tau = \tfrac{1}{8}\Delta T^2 = 0{\cdot}1019$ s^2.

TABLE 10. VALUES OF GRAVITY AND SLIT POSITIONS

set	z_1 (μm)	z_2 (μm)	z_3 (μm)	z_4 (μm)	$-\tau\delta g$ (μm)	ξ_a (μm)	ξ_b (μm)	δg (mgal)
A	−62·54	−54·81	+60·00	+53·35	−1·00	−57·67	−3·60	+0·98
B	−61·84	−55·85	+60·05	+52·87	−1·20	−57·65	−3·29	+1·18
C	−62·67	−54·53	+60·37	+53·28	−0·89	−57·71	−3·81	+0·87
mean	−62·35	−55·06	+60·14	+53·17	−1·03	−57·68	−3·57	+1·01
P	−63·21	−55·58	+59·81	+52·68	−1·58	−57·82	−3·69	(+1·53)

$g = \delta g + 981180{\cdot}00$ mgal, $\tau = \tfrac{1}{8}\Delta T^2 = 0{\cdot}1019$ s^2.
Calculated values of z(μm): 1, −62·28; 2, −55·14; 3, +60·22; 4, +53·08.

Each complete set of observations in the four configurations gives four observation equations to be solved by least squares for δg, ξ_a and ξ_b. The results are

$$-\tau\delta g = \tfrac{1}{4}(z_1+z_2+z_3+z_4),$$
$$\xi_a = \tfrac{1}{4}(z_1+z_2-z_3-z_4),$$
$$\xi_b = \tfrac{1}{4}(z_1-z_2-z_3+z_4).$$

Table 10 contains the values of z, δg, ξ_a and ξ_b for the separate sets and for the overall mean. The values for sets A, B and C are all quite consistent, as are the values of ξ_a and ξ_b for set P, but the value of δg for set P is significantly different because of the unknown timing errors in this set.

The residuals are summarized in table 11.

Table 11. Summary of residuals

(Mean residuals of z(μm))

set A	+0·03	configuration 1	−0·07
B	−0·16	2	+0·08
C	+0·14	3	−0·08
		4	+0·09

variance of single residual	0·187 (μm)^2	on 9 degrees of freedom.
s.d.	0·43 μm	
standard deviation of g	0·12 mgal	

Certain corrections have to be applied to the measured value, 981181·01 mgal, of the apparent acceleration.

In the first place, the length used in calculating g is that as measured with the invar frame surrounded by air at atmospheric pressure whereas the times are measured with the frame under vacuum and therefore longer by

$$\delta l = \tfrac{1}{3}lp/K,$$

where K is the incompressibility of invar, l is the length of the frame and p is $10^5 N/m^2$. The value of K is discussed in appendix 1 and is taken as

$$1{\cdot}1\times 10^{11} N/m^2, \quad \text{s.d. } 0{\cdot}14\times 10^{11} N/m^2.$$

Hence $\delta g = g\delta l/l = 0{\cdot}25$ mgal, s.d. 0·03 mgal.

The effect of the compression of the slit blocks is less than 0·01 mgal.

Secondly, the measured value is corrected to the position of the lower slit by the formula given at the end of §2, γ being $3\times 10^{-6}/\text{s}^2$:

δg is 0·20 mgal with a negligible uncertainty.

Thirdly, the value is corrected to the floor of the laboratory using the measured gradient of gravity (§2). The lower slit is 1·08 m above floor level and so

$$\delta g = 0{\cdot}34 \text{ mgal}, \quad \text{s.d. } 0{\cdot}01 \text{ mgal}.$$

Lastly, there is a very small correction of 0·02 mgal for buoyancy forces at the mean working pressure of 15 μb.

The value of gravity at floor level in the laboratory is therefore 981181·82 mgal.

Finally, for comparison with other observations the value at the British Fundamental Station is required; using the difference of −0·07 mgal (§1) it is 981181·75 mgal.

(c) *Units of length and time*

The measurements of length depend on the wavelengths of the lines in the spectrum of krypton and mercury used in the interferometric measurements. The full measurements of block separation are referred to the conventional wavelengths for the green and violet lines of the spectrum of mercury-198 *in vacuo*, namely 5462·2706 Å† and 4359·5625 Å respectively.

The wavelength of the green line from the particular lamp used was compared with the krypton standard line (λ 6056 Å) by Dr W. R. C. Rowley, the result being 5462·27065 Å, with a standard deviation of 1 in 10^8. Use of the conventional values should therefore not introduce an uncertainty of more than 0·01 mgal.

The check measurements depend on the length of the 20 cm interferometer as measured with lines from a krypton-86 lamp run under the standard conditions and therefore with wavelengths within 1 in 10^8 of the standard values used to define the metre.

The practical unit of length in this determination is therefore within 1 in 10^8 of the metre defined as equal to 1 650 763·73 wavelengths of the orange line (λ 6056 Å) emitted by krypton-86 under specified conditions.

The standard 100 kHz signal on which the timing depends was obtained from the rubidium gas cell working standard oscillator of the N.P.L., the frequency of which was always within 1 in 10^{10} of 100 kHz in terms of the atomic unit of time in which the frequency of the caesium-133 hyperfine line at zero field is 9 192 631 770 Hz.

11. Errors and comparisons

(a) *Discussion of errors*

The majority source of the uncertainty of the final value of gravity is the scatter of the observed times which mainly arises from the long-period microseismic disturbances: it was shown in § 10 (*b*) that the observed scatter of the times is very closely that to be expected from the estimated errors of reading times from the photographs and from the estimated spectrum of long-period microseisms.

The mean value of the variance of the average value of a set of ΔT^2 values is

$$0{\cdot}0884 \times 10^{-12}\,\mathrm{s}^4$$

(table 7) which is equivalent to 0·134 (μm)^2.

The uncertainties of the measured lengths arise mainly from the changes between the measurements before and after the time measurements. The standard deviation of a single measurement of length, as estimated from the differences between the observed and calculated orders of interference in the 20 cm interferometer, is less than 0·1 of an order in the 20 cm interferometer or about 0·1 μm in the separation of the blocks, whereas the standard deviation estimated from differences between full and check measurements is 0·24 μm from all observations of sets *A*, *B* and *C* but 0·13 μm if the two largest differences are omitted. The two largest differences must have arisen from real changes in the positions of the blocks but the other discrepancies seem consistent with the errors of measurement.‡

† 1Å = 10^{-10} m exactly.

‡ The effects of errors in estimating the refractive index of the air are considered separately in Appendix 2.

The standard deviation of the mean of two measurements of the separation of the blocks before and after time measurements is estimated to be 0·17 μm.

Combining the estimates of the variances of the mean value of a set of time measurements and of a pair of length measurements, the variance of a single value of z is expected to be 0·163 (μm)^2; the value actually found is 0·187 (μm)^2. The agreement is quite satisfactory and indicates that the principal sources of random error have been correctly identified.

The calculated values of the position of the slit planes within the blocks are a useful indication of the consistency of the measurements and of gross errors and as may be seen from table 10, the values of ξ_a and ξ_b are very consistent; the values from set P agree well with those from the other sets as would be expected from the fact that the timing error in set P is a systematic one affecting the whole set. The mean residuals for the sets and configurations (table 11) are not large and do not suggest any significant difference between sets or configurations.

It is believed that the design of the experiment and the reversal and interchange of the blocks have eliminated systematic errors arising from imperfections in the construction and alinement of the blocks, time constants of the timing system and similar errors. The environment affects the measured acceleration through the air resistance and electrostatic charging; the uncertainty of the experimental determination of the pressure effect corresponds to about 0·03 mgal at the mean pressure of the experiments but the theoretical basis of the prediction that the effect should be zero seems quite secure at this pressure and there seems no need to allow for pressure effects in estimating the overall uncertainty of the value of gravity. The effect of electrical charging would be variable instead of constant as any air pressure effect would be and so far as it is random with a zero mean, the variability it produces is included in the observed scatter of the times. The question is whether the overall uncertainty of the final value of gravity should be increased to allow for a possible systematic error on this account. It has been argued above that the fact that no dependence of acceleration on air pressure has ever been observed at the pressures of observation can be taken as evidence that electrostatic charging under the conditions of the determinations has no systematic effect within the errors of observation and when the fact that no charging has been observed by direct observation at these pressures is also considered, it seems justifiable to make no allowance in the overall uncertainty for possible charge effects. A possible effect of charge on the slit blocks is negligible (appendix 3).

The observations of times are on the average made at temperatures 0·22 degC less than the corresponding measurements of length; the standard deviation of 0·05 μm/degC in the coefficient of expansion of the invar frame therefore induces an uncertainty of 0·01 mgal in the final value of gravity.

The gradient of temperature along the invar frame also differs slightly between the measurements of time and of length, the average differences being: set A, 0·00; B, +0·09; C, +0·08 degC, the positive sign indicating that the gradient is greater during the timing measurements.

It was shown in §8 that there appeared to be a systematic dependence of the length of the frame on the gradient of temperature, presumably because the five thermocouples do not give a true average temperature. The numerical value corresponds to −1·1 mgal/degC,

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the length decreasing as the gradient increases. There is some indication of the same effect in the observed differences between full and check observations of the block separation, which can be represented by $\delta L = 0{\cdot}18\,\mu\text{m} - 1{\cdot}1\delta\theta$ ($\delta\theta$ is the difference of gradient (degC)), with a standard deviation of about $0{\cdot}25\,\mu$m. However, this result depends essentially on the three or four largest differences between full and check measurements and the mean value is not zero as would be expected if the differences were just due to gradient differences. It is clear that real changes in block positions have occurred, and these data do not fully support the other observations. Furthermore, there is no correlation between values of the gradient differences and the residuals of z, either for individual values or for the mean of sets. The observations of the gradient effect were made when the temperature was rising rapidly and it is possible that they are not applicable to the much more stable conditions under which the definitive measurements of gravity were made. No correction will therefore be applied to the measured value of gravity on this account but the standard deviation will be increased by a contribution of 0·04 mgal.

The errors of alinement of the whole apparatus with respect to the horizontal (§2) are not entirely negligible. The random part is already included in the scatter of the values of z but there could be a systematic error of about 0·02 mgal, the measured value of gravity being too small since which ever way the slit blocks depart from the horizontal, the measured separation always exceeds the true vertical distance. On the other hand, the measured separation is systematically too great because of the cosine effect in the white light interferometer system (§4) and it happens that this nearly cancels the other error. No separate provision is therefore made for these errors of alinement.

Finally, the uncertainty of the thickness and form of the slit blocks is common to all measurements and contributes 0·02 mgal to the overall standard deviation.

The following list summarizes the contributions to the overall uncertainty of the value of gravity at the floor of the laboratory:

residuals of length and time measurements	0·12 mgal
compressibility of invar	0·03
coefficient of expansion of invar frame	0·01
differences of temperature gradient	0·04
reduction to floor level	0·01
total	0·13

The uncertainty of the value transferred to the British Fundamental Station is effectively the same.

The final result is therefore

$g =$ **981 181.82 mgal** at the floor of the laboratory in Bushy House,

$g =$ **981 181.75 mgal** at the British Fundamental Station;

The standard deviation of each value is 0·13 mgal.

(*b*) *Comparisons with other observations*

Clark (1939), using a reversible pendulum, made an absolute determination of gravity in the room containing the British Fundamental Station and his result, reduced to the fundamental station, is

$$g = 981\,183{\cdot}2\text{ mgal},\quad \text{s.d. } 0{\cdot}7\text{ mgal}.$$

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The difference from the present result is 1·4 mgal, just twice the standard deviation of Clark's value. The uncertainty of Clark's result arises mostly from changes in the period of the pendulum according to its location of the knife-edge support and since there is no assurance that the mean value of such knife-edge effects should be zero, the real uncertainty of Clark's result could be greater than is indicated by the standard deviation.

Comparisons with other determinations depend on measurements of the differences of gravity between the respective sites. Very few such differential measurements have uncertainties as small as that of the present determination and the comparisons are thus on the whole not very informative. The most reliable is the one between the N.P.L. and the Bureau International des Poids et Mesures at Sèvres, where a free-fall experiment with a graduated scale has been carried out by Thulin (1961). The difference between Sèvres Point *A* and the British Fundamental Station is

255·71 mgal, s.d. 0·06 mgal.

Thulin's measured value of gravity at Sèvres Point *A* is

980928·0 mgal, s.d. 0·7 mgal;

reduced to the British Fundamental Station it is

981183·71 mgal, s.d. 0·7 mgal,

which is 1·9 mgal, or about three times Thulin's standard deviation, greater than the present value. Such a large discrepancy would not be expected from the care with which Thulin worked.

The determination by Preston-Thomas and his colleagues at Ottawa (1960) suggests that there are unsuspected sources of error in falling bar experiments, for in that work two similar bars were used and gave values of gravity that differed by 1·5 mgal. It is more difficult to compare this result with the present one because of uncertainties in the measured differences of gravity across the Atlantic (Cook 1965*a*). The difference between the values of gravity at the optic axis of the Ottawa apparatus and at the British Fundamental Station is

−569·13 mgal, s.d. 0·3 mgal;

the absolute value in Ottawa was

980613·2 mgal, s.d. 1·0 mgal;

and the difference from the present value is therefore +0·6 mgal which is well within the uncertainties of the observations and the comparison. Faller (1963) has determined gravity at the University of Princeton by a free-fall experiment in which the falling object was one reflector of an optical interferometer. The uncertainty of his result is dominated by the rather large effects of microseisms resulting from the fact that the entire measurement was completed in 5 cm; his value is

980160·4 mgal, s.d. 0·7 mgal,†

at a site where the value of gravity is lower than that at the British Fundamental Station by

1021·4 mgal, s.d. 0·4 mgal.

The difference from the present value is therefore effectively zero.

† A suggested change in the correction for eddy current damping (Cook 1965*a*) is incorrect.

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These comparisons and others of which details are given by Cook (1965*a*) are listed in table 12. One would like to use them to detect systematic errors in the various classes of experiment but the random errors are too great for that to be possible; however, the following comments may be permitted. Pendulum and free-fall experiments with graduated scales give internal evidence of systematic errors. The differences from the present value are mostly not inconsistent with the uncertainties assigned to the various determinations. The mean of all results apart from the old Potsdam one is 0·46 mgal greater than the present value and, subject to doubts about the transatlantic comparisons, the present value could be taken to represent our knowledge of the absolute value of gravity with an uncertainty of not more than 0·5 mgal.

TABLE 12. COMPARISONS OF ABSOLUTE DETERMINATIONS OF GRAVITY

site	reference	difference from present value (mgal)	s.d.
N.P.L.	Clark (1939)	+1·4	0·7
Sèvres A	Thulin (1961)	+1·9	0·7
Ottawa N.R.C.	Preston-Thomas *et al.* (1960)	+0·6	1·0
Princeton	Faller (1963)	0·0	0·8
Leningrad V.N.I.I.M.	Yanovskii (1958)	+1·5	1·0
Washington N.B.S.	Heyl & Cook (1936)	−2·2	1·3
Potsdam	Kühnen & Furtwängler (1906)	+13·7	—

The s.d. is the combined standard deviation of the absolute measurements and the comparisons. For details of measurements and comparisons, see Cook (1965*a*).

12. CONCLUSION

The present determination has verified the expectations that the symmetrical free motion experiment would be essentially free of systematic errors and that, in particular, it would not be afflicted by air resistance. The expected effects of microseisms have also been observed, and a better result could have been obtained if the operation of the apparatus had been more reliable so that time could have been given to recording the microseisms during the flight of the ball. The difficulties in obtaining a reliable trajectory for the ball, like those arising from electrostatic charging, stem from the problems of operating the catapult in a good vacuum and set the ultimate limit to the accuracy of this determination. Were these problems to be overcome, the present apparatus is clearly capable of appreciably higher accuracy.

Almost all the preliminary experimental work and the development of the final apparatus was done jointly with Mrs H. M. Dale (Miss H. M. Richardson) and the success of the determination owes much to her skill and insight. The measurements would have been impossible without very careful design and construction of the apparatus. Mr L. F. Wilson was responsible for the design work and the mechanical parts were constructed in the East Workshop of the N.P.L. by Mr W. Goddard and Mr G. Long. Both the first glass and second silica slit block assemblies were made in the optical workshop of the Light Division by Mr Cooper and Mr Sussex under the supervision of Mr R. Peters. Mrs V. P. Matthews, Mrs M. Grove and Mrs S. Swan have assisted in some of the observations and calculations. I am

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indebted to Dr J. Faller and Dr L. Essen F.R.S., for comments on the manuscript of this paper.

The determination has formed part of the programme of general research of the National Physical Laboratory and this paper is published with the permission of the Director.

Appendix 1. The bulk modulus of invar

The bulk modulus of invar does not appear to be well known. Two values are given in the literature and there are also values of Young's modulus and estimates of Poisson's ratio. The two values of the bulk modulus are quoted by Marsh (1938) and were obtained by Mehl and by Ebert & Kussman for 36% nickel steel. Of the values for Young's modulus given in the literature, the only one with a stated origin is that that depends on the measurements of Guillaume and it is not clear whether the others are independent. Young's modulus has a minimum value at about the composition of invar. The value of Poisson's ratio used in converting values of Young's modulus to bulk modulus is 0·31, as given by Hoyt (*Metals and alloys data book*, 1943); Poisson's ratio for iron is 0·29, the range quoted for nickel is 0·29 to 0·33 and for constantan, with 40% nickel, the ratio is 0·33. The available data are summarized below (K and E in 10^{11} N/m^2)):

source	E	K	equivalent K
Mehl	—	1·16	—
Ebert and Kaussman	—	0·79	—
Guillaume (*Circ. Nat. Bur. Stand.*, 58, C 447, Hoyt 1943)	1·45	—	1·27
Lyman 1961	1·47	—	1·29
Kaye & Laby (13th ed.) 1966	1·44	—	0·99

For constantan, $K = 1{\cdot}57$, $E = 1{\cdot}63 \times 10^{11}$ N/m^2; the values for invar will be less than these.

The value of K for invar adopted in the paper is $1{\cdot}1 \times 10^{11}$ N/m^2 with a standard deviation of $0{\cdot}14 \times 10^{11}$ N/m^2.

Appendix 2. Effects of errors in estimating the refractive index of air

Errors in estimates of the refractive index of air affect the observations in three ways that may be significant:

they cause errors in the lengths of the 20 cm interferometer obtained from the photographic observations;

they lead to discrepancies between the order of the 20 cm interferometer observed photoelectrically and that calculated from the measured length and estimated refractive index;

they introduce errors in the relation between the length of the 20 cm interferometer and the separation of the slit blocks;

they cause errors in the calculated separation of the slit blocks.

The first two effects contribute to the discrepancies, but do not cause errors in the slit block separation if that is derived from the observed order. The final two effects do introduce errors in the estimates of the slit block separation.

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Errors in the estimated refractive indices arise when the actual values of pressure, temperature, humidity and content of carbon dioxide within the interferometers differ from the values of pressure, temperature or humidity measured outside the interferometers or from the content of carbon dioxide usually assumed (0·03%). The discrepancy in pressure may be ignored. The air in the 20 cm interferometer would be the more likely to have an humidity different from that of room air, but care was taken to mix the air in the piston frequently with that in the room and the observations show no evidence of differences of humidity.

The actual carbon dioxide content does appear to have differed somewhat from the assumed value of 0·03 %. Although the room was well ventilated, observations show that the carbon dioxide content may have doubled during the course of a morning or an afternoon in which observations were being made. Such a change would not show in the measurements of the 20 cm interferometer since that was almost always closed to the room air after the room had been unoccupied (but ventilated) for some time. (The good agreement between the observed and calculated orders in the 20 cm interferometer shows that the assumption of 0·03 % carbon dioxide is satisfactory for the well ventilated room at the start of a series of observations). The change in carbon dioxide content would however, show in the differences between full and check measurements (table 6) since in rather more than half of the pairs of such measurements, the full measurement was made when the room had been unoccupied for some time whereas the check measurement was made after the room had been occupied for observations for an hour or so. The check measurement is on the average greater than the full measurement, as would be expected if the amount of carbon dioxide had increased. On the other hand, the differences are no greater than expected from the known uncertainties of the separate measurements, there is no clear correlation in the data between the observed difference length and the estimated difference of carbon dioxide content, and the largest discrepancy between full and check measurements occurred when the estimated carbon dioxide content was greater for the full than for the check measurement. No effect of change of carbon dioxide content can therefore be detected in the data. If, however, it were supposed that the differences between full and check measurements were the result of a change in carbon dioxide content, the mean measured lengths should be reduced by 0·05 μm and the value of g by 0·05 mgal.

Temperature gradients in the air between the slit blocks may mean that the average temperature read by the three thermocouples hung in the air between the blocks is not the average temperature of the air. A somewhat different average is given by the five thermocouples attached to the invar frame and slit blocks; the mean average 'air' temperature differs insignificantly (0·015 degC) from the mean average 'frame' temperature under conditions in which the differences between upper and lower air temperatures range from 0·23 to 0·72 °C. It is considered therefore that no significant systematic error arises from incorrect averaging of the air temperature although fluctuations in conditions may contribute to the random scatter of results. To summarize, it is possible that uncertainty of the carbon dioxide content may have led to the measured lengths being systematically 0·05 μm too great but this is thought unlikely.

The values given by Edlén's (1953) refractive index formula, used in all the reductions, have standard deviations of about 3×10^{-8}, an uncertainty small compared with the other errors of the determination of g.

Appendix 3. Attraction of a charged slit block on a conducting ball

Charge on the lower block has no effect on the ball because it is shielded by the metal plate, tube and cone placed on the block to protect it from a deflected ball; the plate is in contact with the earthed metal cage of the blocks and is effectively earthed.

Charge on the upper block is partially, but not completely, shielded by the metal cage holding the block and by the invar plate supporting it. The distribution of charge is unknown but it must be outside the hole in the block; if it is close to the edges of the block it will be shielded by the cage and plate, and so the worst distribution seems to be a ring of charge around the central hole in the block. Let it be represented by a uniform distribution around a ring of the same diameter as the central hole. By considering the image of the ring of charge in the conducting ball, the force on the ball may be shown to be closely

$$Q^2rz/(R^2+z^2)^2,$$

where Q is the total charge, z is the distance of the centre of the ball from the plane of the ring, R is the radius of the ring and r is the radius of the ball.

If m is the mass of the ball, the acceleration is $Az/(R^2+z^2)^2$ where $A = Q^2r/m$ and it may be shown that

$$\frac{\delta g}{g} = \frac{4}{3}\left(\frac{1}{2g}\frac{A}{R^3}\right)\left(\frac{h}{H}\right)^{\frac{1}{2}},$$

where h is the height the ball rises above the upper slit block and H that above the lower.

No great reliance can be put on the numerical values but the form and order of magnitude of this result should be reliable.

The numerical factor $\frac{4}{3}(h/H)^{\frac{1}{2}}$ is about $\frac{1}{5}$.

If the charge is about 1 e.s.u., $A/2gR^3$ is about $1/10^8$, giving an estimated error of 2×10^{-9} g.

It is difficult to estimate the charge. 1 e.s.u. corresponds to about 100 V and if the voltage were 1 kV, the error would still be only about 2×10^{-7} g.

Such large voltages are most unlikely. The block is in a metal cage and after being turned into a new configuration, when it might acquire charge, it is left in humid air at atmospheric pressure for a few hours. The surface resistivity of fused silica in air of 50% humidity is $3\times10^{12}\ \Omega/\text{cm}^2$ and the leakage resistance will therefore be about $10^{12}\ \Omega$. With a capacity of about 10 pF, the time constant is 10 s and thus even if the block were initially charged to a few kilovolts, the voltage would fall to a quite insignificant value after an hour or so.

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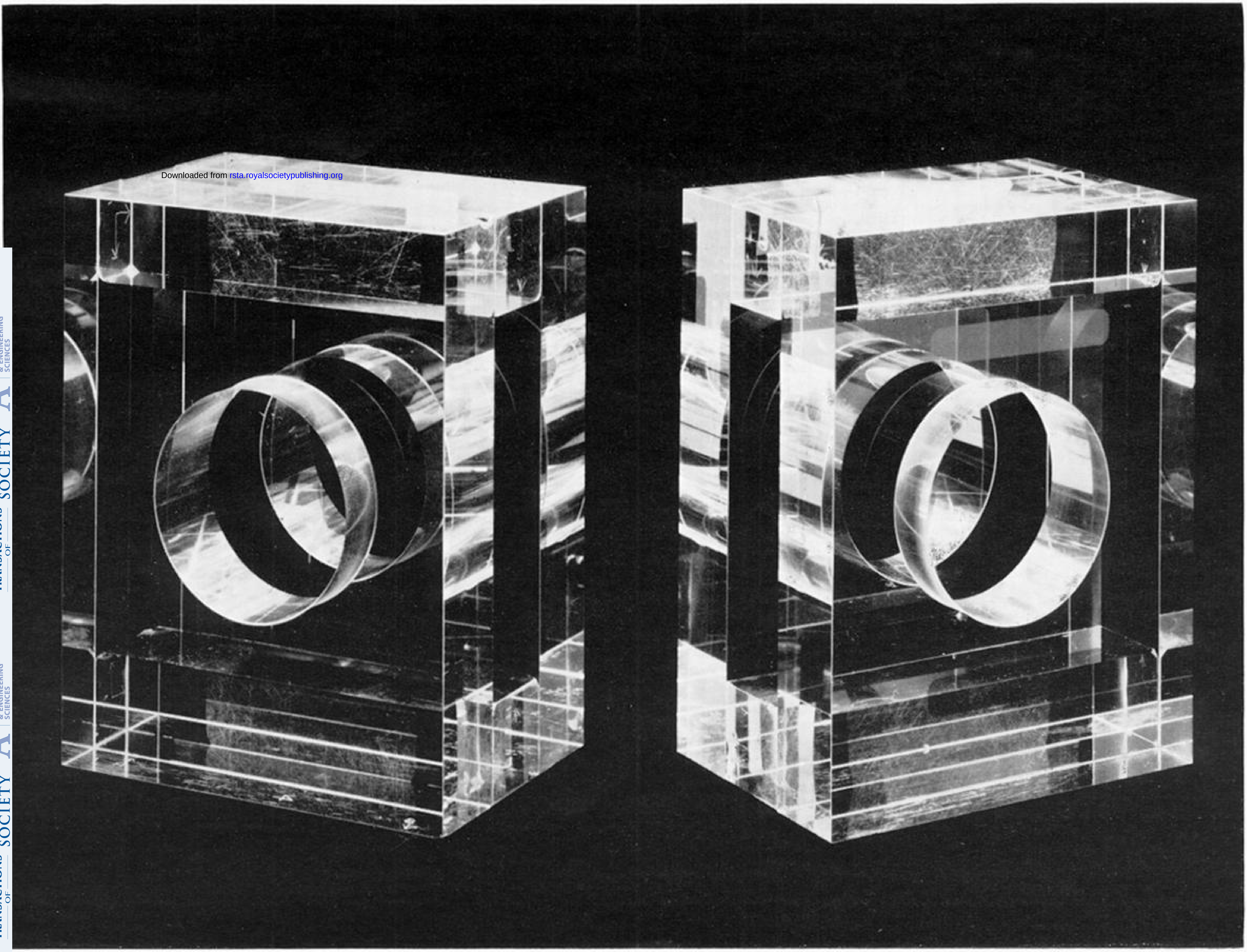

FIGURE 5. Photograph of slit blocks.